What is a force?

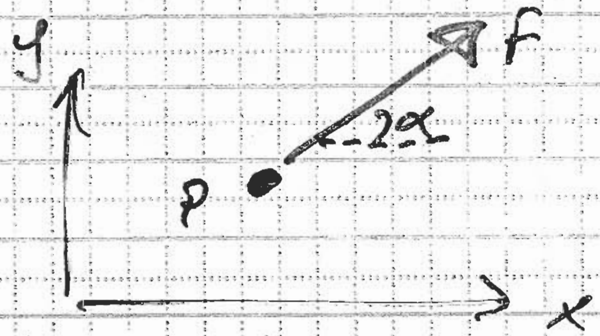
A force is a vector quantity that has both magnitude & direction.

By Newton's 2nd Law,

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$F = m \times a$$

Units $F\ (N) = \underset{(m)}{kg} \times \underset{(a)}{\frac{m}{s^2}}$



This force F, acts on P at an angle of α to the x-axis

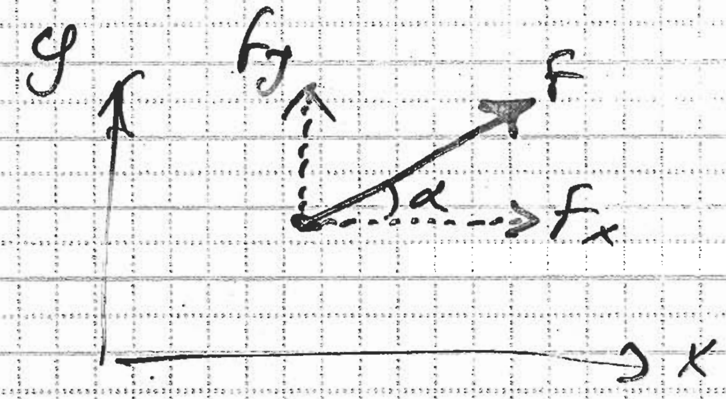
i.e. it has both magnitude (F) and direction (α).

Components of a force

The force acting on P can be replaced with any number of forces we wish, once their combined action (called the resultant) is the same as F.

Usually we are only interested in two components:

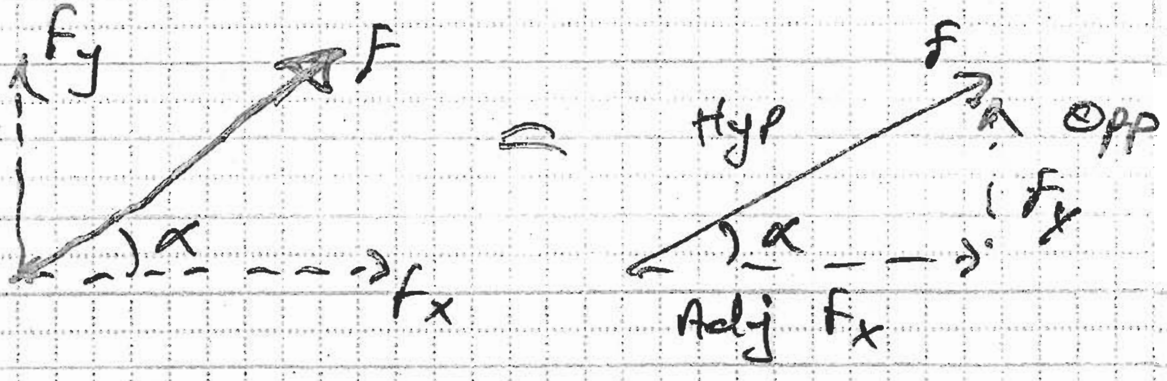
- a force parallel to the x-axis, F_x
- a force parallel to the y-axis, F_y



F_x is the amount of F that acts along the x-axis.

Similarly for F_y.

Using trigonometry:

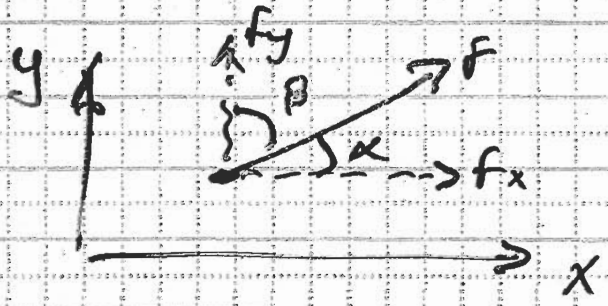


$$S = \frac{opp}{Hyp} \quad C = \frac{adj}{hyp} \quad T = \frac{opp}{Adj}$$

So, $\sin \alpha = \dfrac{F_y}{F}$ $\quad \cos \alpha = \dfrac{F_x}{F}$

$\Rightarrow$ $\boxed{F_y = F \sin \alpha}$ $\quad \boxed{F_x = F \cos \alpha}$

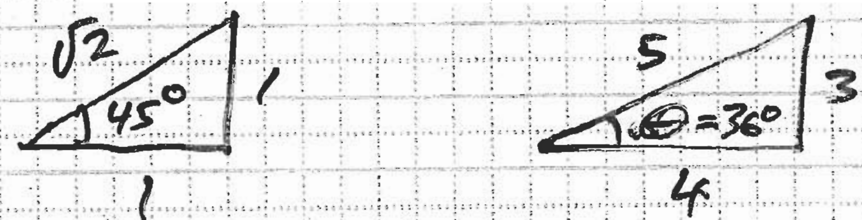
If instead we consider the angle, β:



$$F_x = F \sin \beta \quad F_y = F \cos \beta$$

But these are not used as often.

In structures we often deal with two triangles:



A force acting at 45° will always have the following components:

$$F_x = F \cdot \cos 45° = F \cdot \frac{1}{\sqrt{2}}$$

$$\rightarrow \boxed{F_x = F/\sqrt{2}}$$

Similarly $\boxed{F_y = F \cdot \sin 45° = F/\sqrt{2}}$

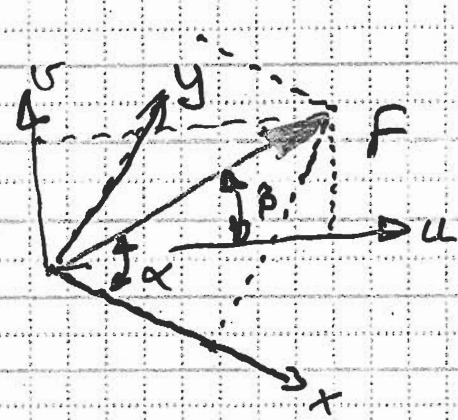
For a force acting at $\theta = 36°$, we have:

$$\boxed{F_x = F \cdot \cos\theta = F \cdot \frac{4}{5}}$$

$$\boxed{F_y = F \cdot \sin\theta = F \cdot 3/5}$$

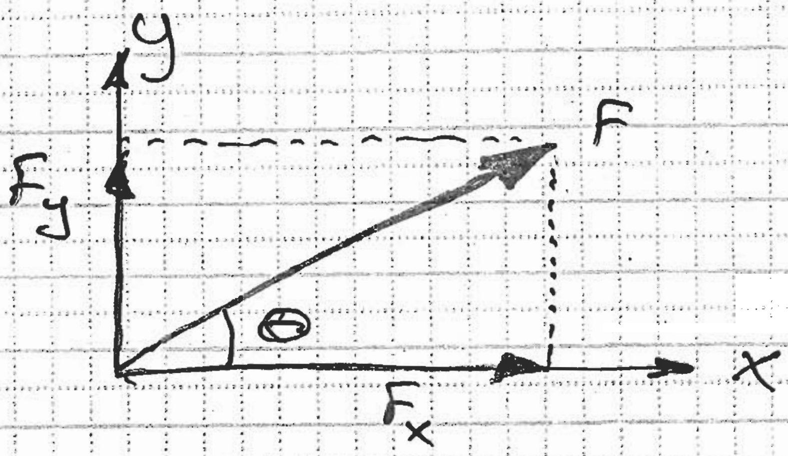
Components of a force

A force is a vector quantity, i.e. it has both direction & size. Often we want to know the size of the force in other directions. We have:



So given any axes system at any arbitrary angle, we can resolve for the components of F along those axes. Above we could do it for the xy axes or the uv axes - whichever we are interested in.

Usually in structures, we are only interested in vertical & horizontal components though. Hence, we have:



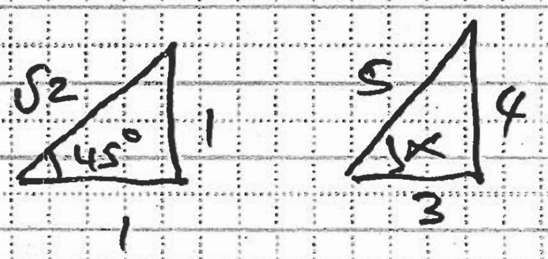
F_y - vertical component of F

F_x - horizontal component of F

$$\sin\theta = \frac{opp}{hyp} = \frac{F_y}{F} \quad \therefore F_y = F\sin\theta$$

$$\cos\theta = \frac{adj}{hyp} = \frac{F_x}{F} \quad \therefore F_x = F\cos\theta$$

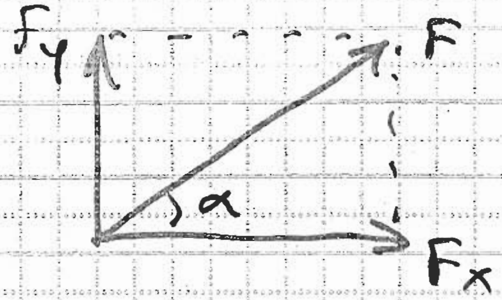
Some easy triangles to work with are:



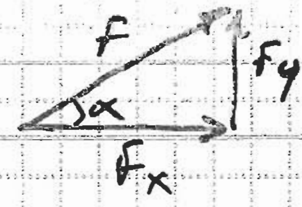
i.e. $\cos 45° = 1/\sqrt{2}$

$\cos\alpha = 3/5$ etc.

Resolving Forces Without Cos or Sin



$\Rightarrow$



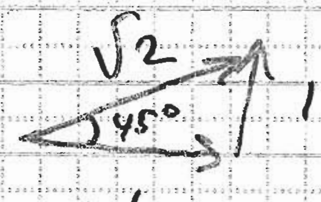
$\therefore \cos\alpha = \left(\frac{Adj}{Hyp}\right) = \frac{F_x}{F}$

$\therefore F_x = F\cos\alpha$

Also, $\sin\alpha = \left(\frac{opp.}{Hyp}\right) = F_y/F$

$\Rightarrow F_y = F\sin\alpha$

If $\alpha = 45°$, then :



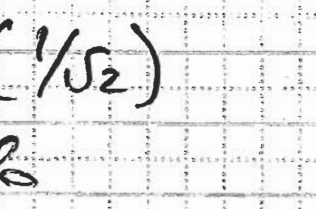
$\therefore \cos\alpha = \left(\frac{A}{H}\right) = 1/\sqrt{2}$

Also, $\sin\alpha = \left(\frac{O}{H}\right) = 1/\sqrt{2}$

$\therefore F_x = F(1/\sqrt{2})$ & $F_y = F(1/\sqrt{2})$

$\therefore F_y = F_x$ for $1-1-\sqrt{2}$ triangle

Another triangle used is

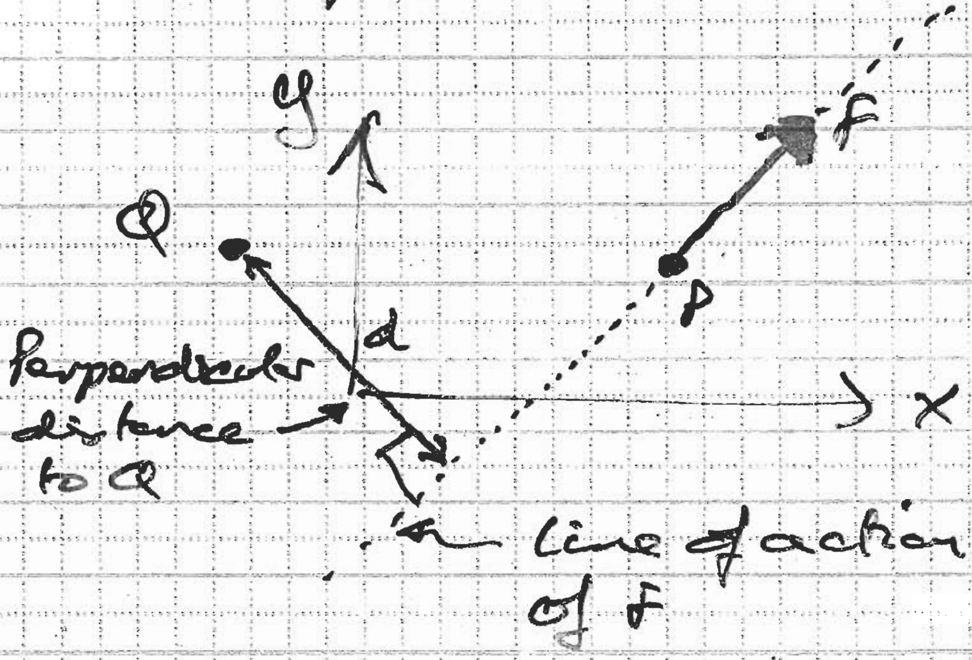


$\Rightarrow F_x = F(4/5)$

$F_y = F(3/5)$

The Moment of a Force

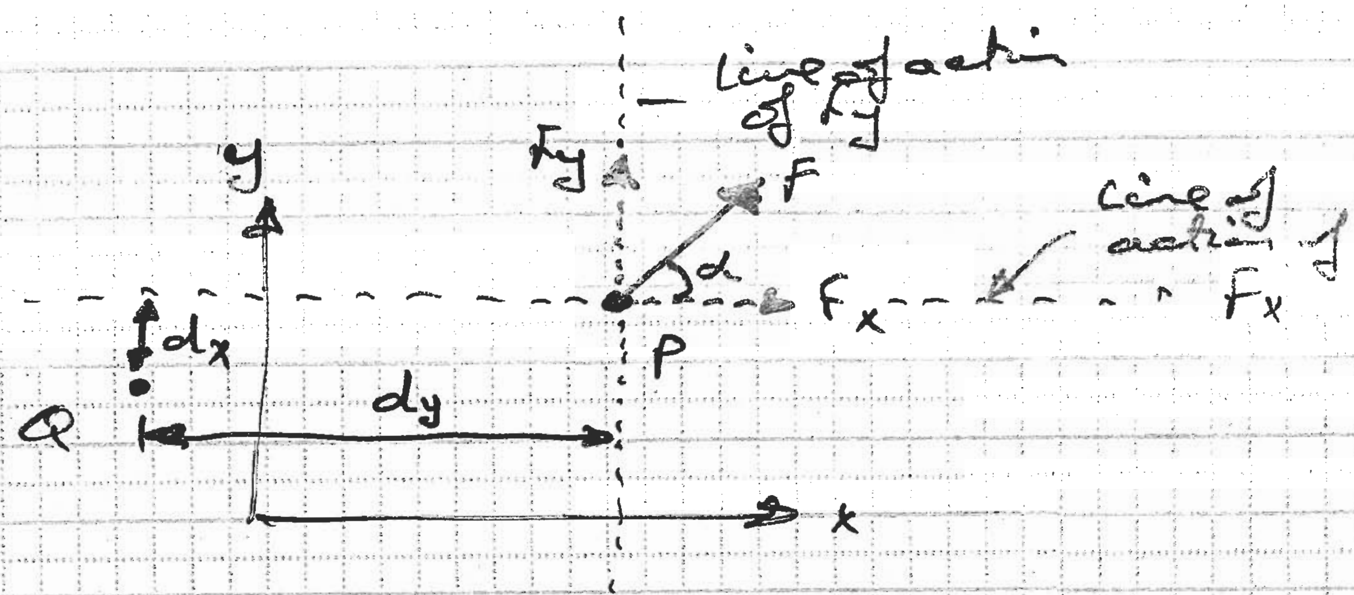
For any point Q, we define the moment of a force (acting at P) to be the product of the force magnitude by the perpendicular distance:



Thus:

The moment of F about Q $= F \times d$

To help us find moments more easily, we can replace F by its components along the x & y-axes:



The moment of F about Q

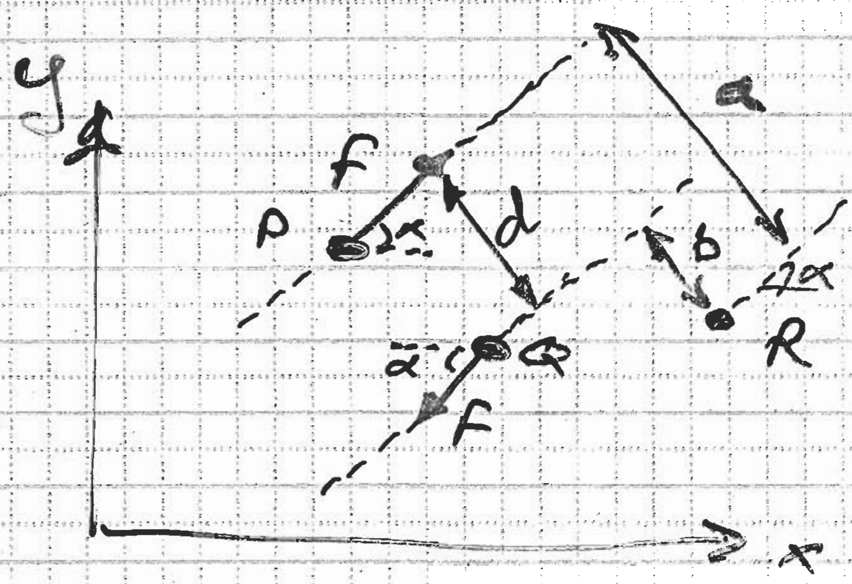
$$= F \times d \quad = F_y \times d_y + F_x \times d_x$$

(as before)

We regularly consider the components of reactions instead of the actual resultant reaction itself. We will see this regularly in the analysis of frames.

A Couple

When two forces, which are equal in size, but opposite in direction, act at a distance d apart, they exert a moment of $M = F \times d$ everywhere on the plane.



The moment of the force F at P about R is:

$F \times a$ — clockwise direction

The moment of the force F at Q about R is:

$F \times b$ — anticlockwise

The total moment is:

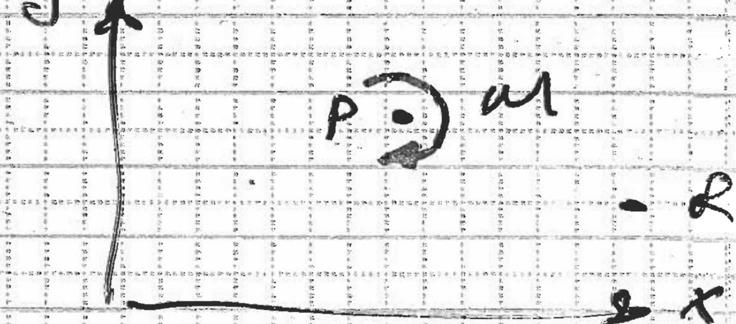
$$M = \underset{\text{ACW}}{F \times a} - \underset{\text{CW}}{F \times b}$$

But, $a = d + b$

$$\Rightarrow M = F(d+b) - F \times b$$
$$= F \times d + F \times b - F \times b$$
$$= F \times d$$

Since a & b can be any value, we have shown that for any point R, anywhere on the plane, the couple causes a moment of $M = F \times d$.

We deal with couples very often and so we usually ignore the forces F, and the distance d and just use the moment M:



Moment of M at P about R = M.
R is any point of the plane.

Equilibrium

- Equilibrium occurs when a body is moving at constant velocity.

  In structures, that constant velocity is zero (we hope!).

- When a body is subject to a change in velocity (i.e. an acceleration), because of its inertia, it feels a force. Newton's 2nd Law states:

  force = mass × acceleration

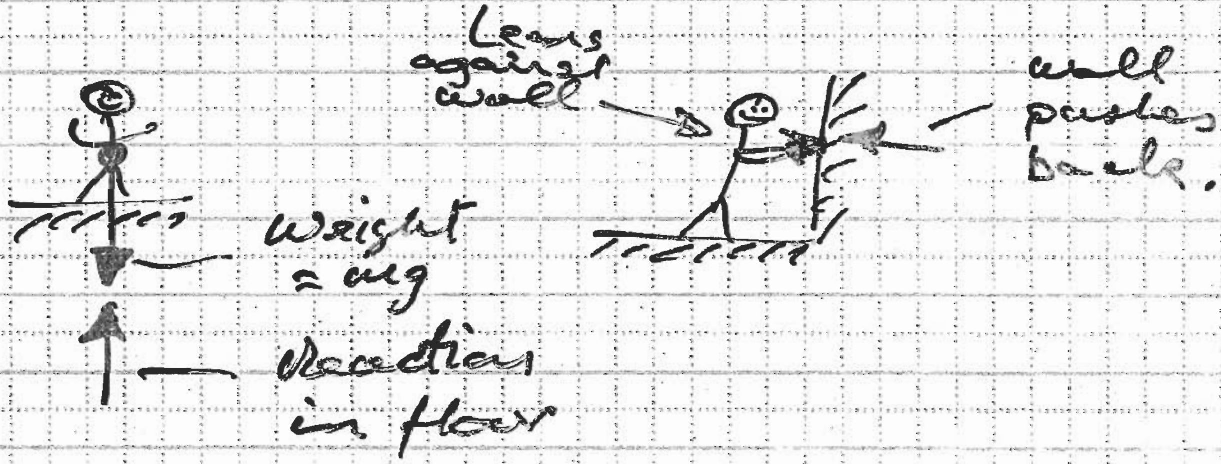
  i.e. $F = m \times a$.

- For a body to be in equilibrium the forces on it must not be imposing an acceleration. Thus, the net forces or moments must be zero.

  For 2-D structures, this gives us 3 equations of static equilibrium:

- $\Sigma F_y = 0$ – Vertical forces
- $\Sigma F_x = 0$ – Horizontal forces
- ΣM about any point $= 0$
  – Moments

Some classic examples of equilibrium are:



Remember:

Mass is measured in kg

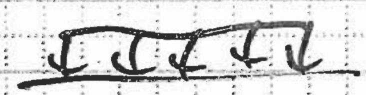
Acceleration in m/s^2

Force in Newtons N.

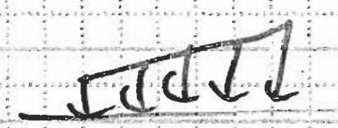
$$\rightarrow N = kg \cdot \frac{m}{s^2}$$

Distributed loads

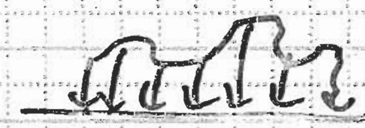
In structures, we do not only have point & moment loads, but also loads distributed along the length of a member:



Uniformly Distributed Loads

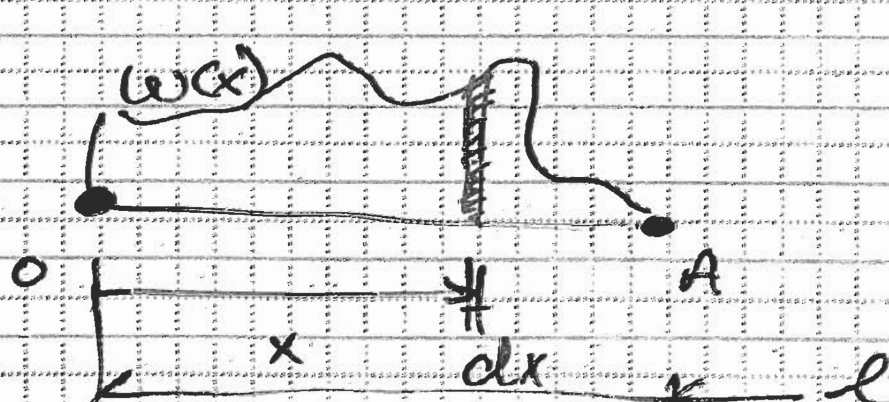


Trapezoidal Distributed Loads



Randomly/Other Distribution of Load.

When taking moments for such loads about a point, we must add up all the little forces x their distances.



Force = $w(x) \cdot dx$

Distance = x

Thus the moment of this small strip of load about O is:

$$\text{moment} = w(x) \cdot dx \cdot x$$

Therefore the total moment about O is the sum of all such little strips:

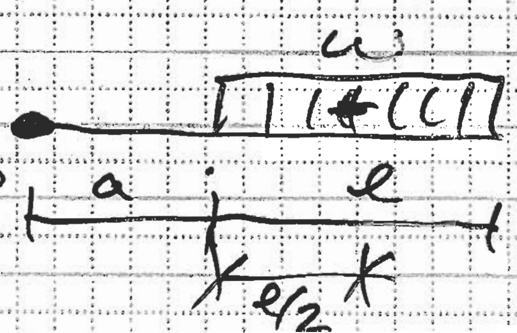
$$M \text{ about } O = \int_0^l w(x) \cdot x \cdot dx$$

$$= \bar{x} \cdot \int_0^l w(x)\,dx$$

$$= \bar{x} \cdot A$$

where $\bar{x}$ is the distance to the centroid of the area of load and A is the area of load i.e. the total amount of load.

For a UDL this is simply:

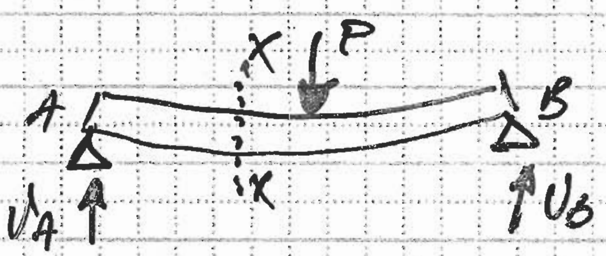


M about O

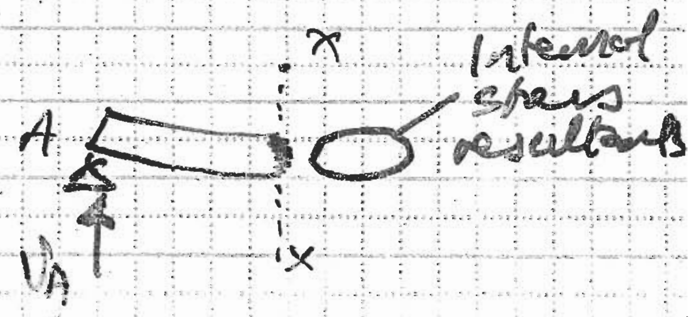
$$= \underbrace{(w l)}_{\text{Total Load}} \times \underbrace{\left(a + \frac{l}{2}\right)}_{\text{Distance to centroid}}$$

Equilibrium of Parts of Structures

We have applied equilibrium to full structures already. However, equilibrium also applies to any part of a structure we wish to isolate:



Beam in Equilibrium



Portion A to X–X is also in equilibrium

When we make a cut and isolate a part of a structure, we need to identify the Internal Stress Resultants that keep the portion of the beam in equilibrium

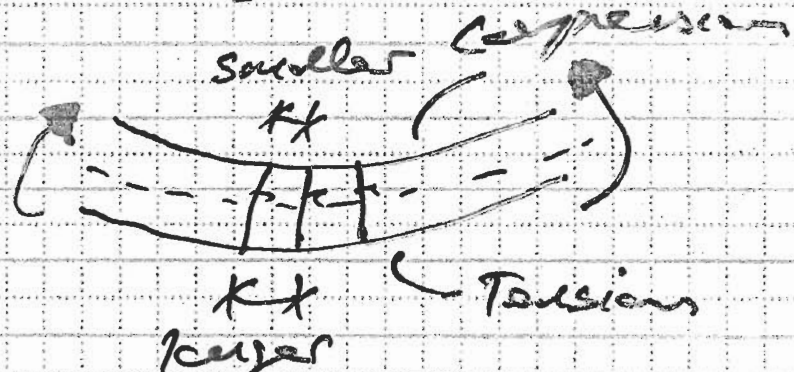
Whenever we make a cut in a structure and identify all of the internal stress resultants and other forces that act on it, we call it a Free Body Diagram

Internal Stress Resultants

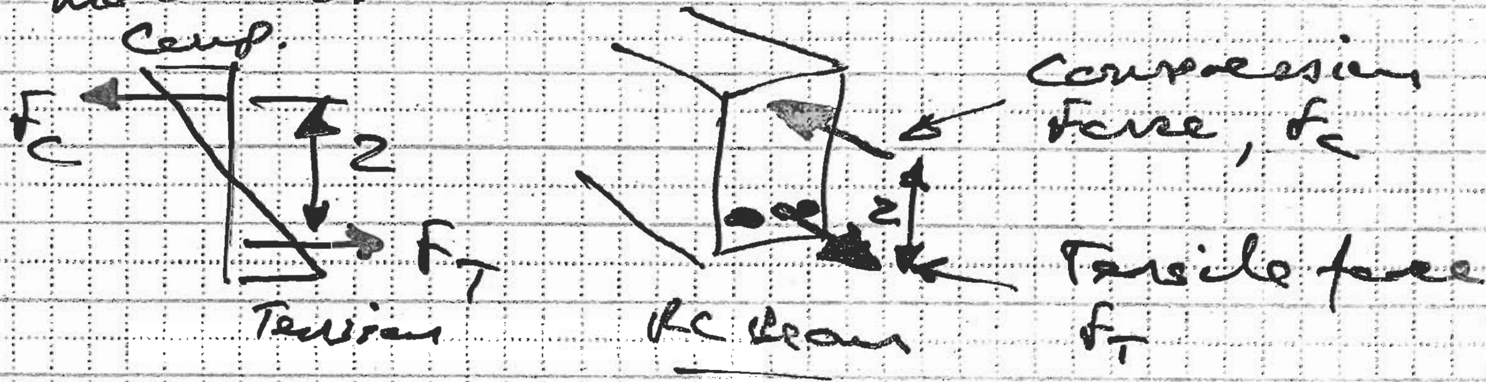
When a material is loaded it deforms. The intensity of loading on a material is stress. The deformation is strain.

- **Bending**

When a beam is bent, it develops tensile and compressive strains (or stresses):

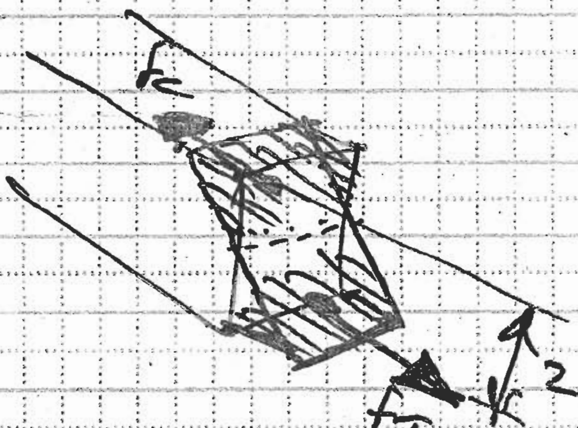


The strains & stresses are linear for usual materials:

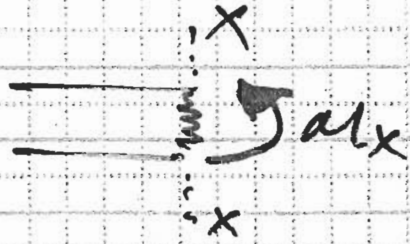


The tension and compression forces are equal & opposite, hence they form a couple, $M = F_T z = F_C z$.

Because these forces form a couple, we can simply identify the effects of internal bending stresses as an internal resisting bending moment.



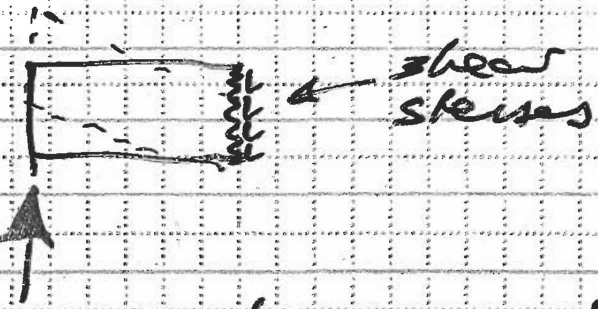
i.e.

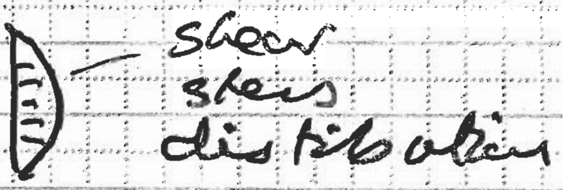


$$M = f_T z = f_C z$$

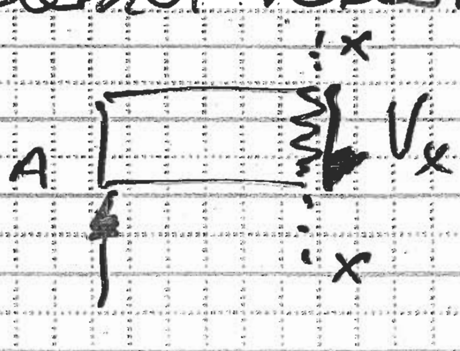
• Shear Forces

When we apply shearing forces to a material it resists these with shear stresses:



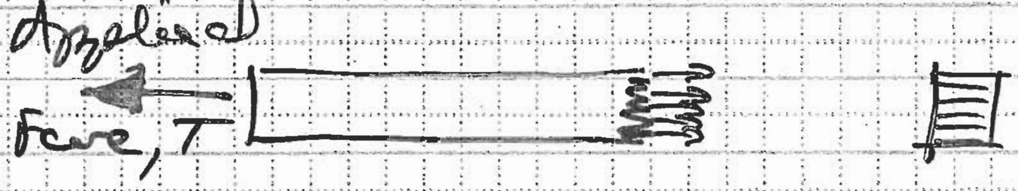


The shear stresses add up to a single internal resisting shear force, V:

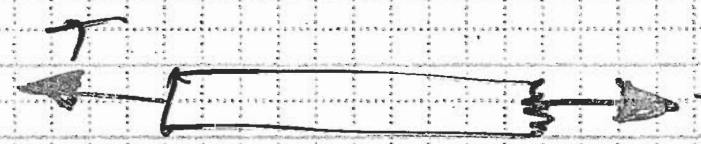


• Axial Forces

When we apply a longitudinal tension or compression force to a member, it resists this by axial stresses:

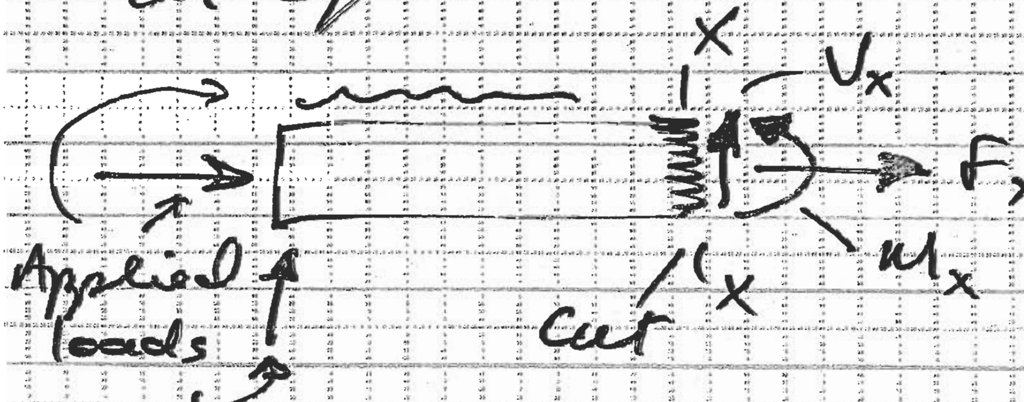


Again these stresses sum to give us an internal resisting force:



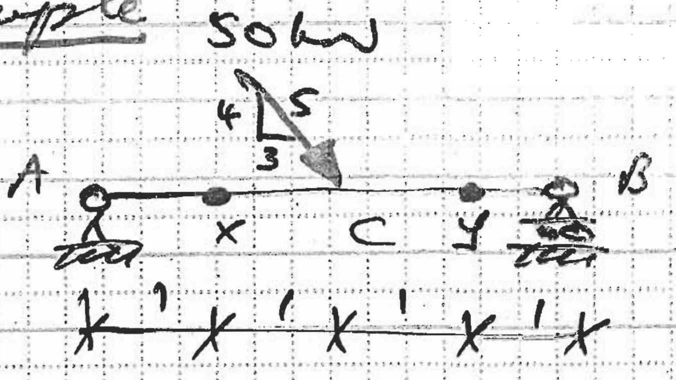
• Summary

Whenever we cut a member (in 2D analysis) we thus have 3 internal stress resultants to help keep it in equilibrium:



Note that some of these may be zero if not reqd.

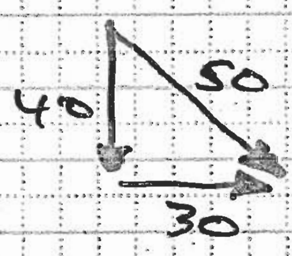
Example

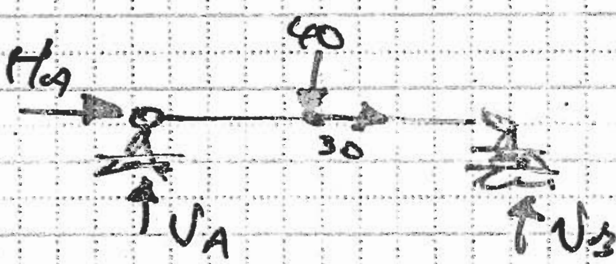


Find the internal stress resultants at X and Y

First we need to find the reactions.

Note that the components of the load are:





$\therefore \Sigma M$ about $A = 0$ +↻

$\therefore 40 \times 2 - 4V_B = 0$

$\therefore V_B = 20\text{kN} \uparrow$

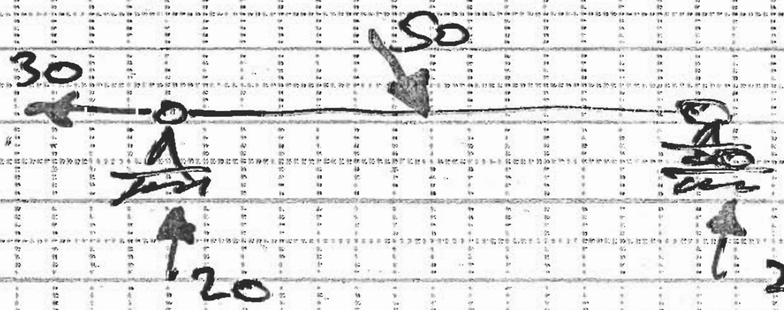
The 30kN force has no ⊥ dist from A.

$\Sigma F_y = 0 \; \downarrow^{+} \therefore 40 - V_A - 20 = 0$

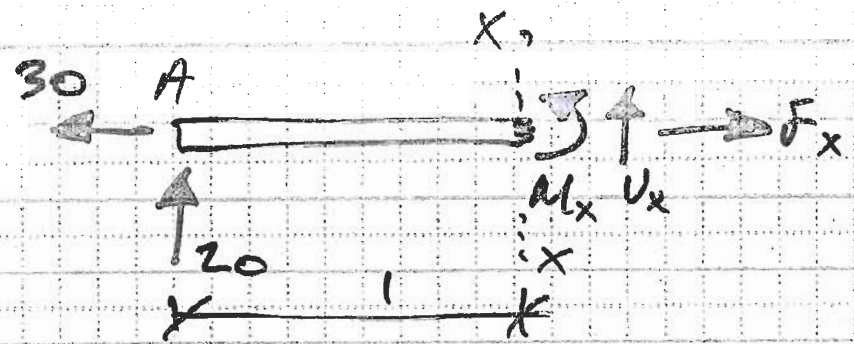
$\therefore V_A = 20\text{kN} \uparrow$

$\Sigma F_x = 0 \rightarrow +$

$\therefore H_A + 30 = 0 \quad \therefore H_A = -30\text{kN}$ i.e. $\leftarrow$



Cut the beam from A to x :



Our 3 eqns of equilibrium can easily be applied to this portion or FBD:

$\Sigma F_x = 0 \overset{+}{\leftarrow} \therefore 30 - F_x = 0 \therefore F_x = 30\text{kN} \rightarrow$

$\Sigma F_y = 0 \uparrow + \therefore 20 + V_x = 0 \therefore V_x = -20\text{kN}$ i.e. $\downarrow$

We can sum moments about any point.

Use A first:

ΣM about $A = 0$ +↻

$\therefore M_x + V_x \times 1 = 0$ But $V_x = -20\text{kN}$

$\therefore M_x - 20 = 0 \therefore M_x = 20\text{kNm}$ ie Tension on btm.

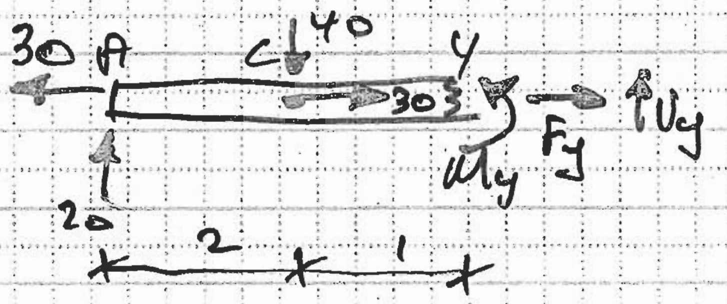
Try x-x to confirm:

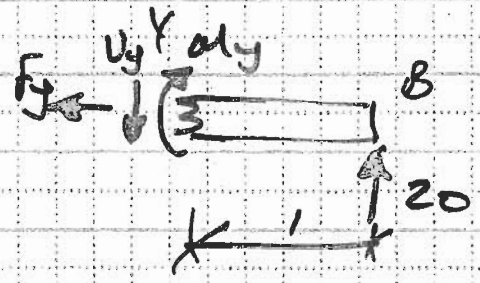
ΣM about $x = 0$ +↻

$\therefore M_x - 20 \times 1 = 0 \therefore M_x = 20\text{kNm}$ as before.

Note: it is usual to take moments about the cut since V_x may not be known.

When finding the values of the internal stress resultants at Y we can take A to Y or Y to B as we please – it never matters which side we choose, we will get the same answer!





$\Sigma F_y = 0 \uparrow +$

$\therefore 20 - 40 + V_y = 0 \quad \therefore V_y = 20\text{kN} \uparrow$

$\Sigma F_x = 0 \rightarrow +$

$\therefore -30 + F_y + 30 = 0$

$\therefore F_y = 0$

$\Sigma M \text{ about } Y = 0 \circlearrowright +$

$\therefore M_y + 40 \times 1 - 20 \times 3 = 0$

$\therefore M_y = 20\text{kNm}$

(+) $\therefore$ Tension on btm

$\Sigma F_y = 0 \uparrow +$

$\therefore 20 - V_y = 0$

$\therefore V_y = 20\text{kN} \downarrow$

$\Sigma F_x = 0$

$\therefore F_y = 0$

$\Sigma M \text{ about } Y = 0 \circlearrowleft +$

$\therefore M_y - 20 \times 1 = 0$

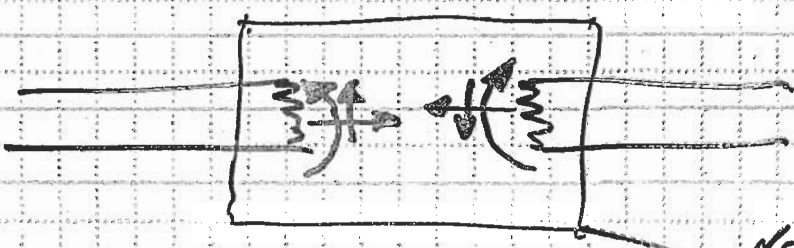
$\therefore M_y = 20\text{kNm}$

i.e. Tension on btm.

So, both methods give the same result but it is clearly easier to take the side with simpler loading, i.e. Y to B.

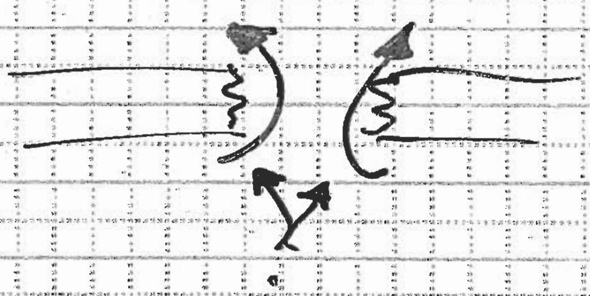
Equilibrium of the Cut

In the example, we saw that we change direction each side of the cut (for the force/moment considered). This is because the cut itself must be in equilibrium:



Released forces and moments in the cut in equilib.

Note also:



Tension is on bottom on both sides as it should

Each side of cut in tension, as it should

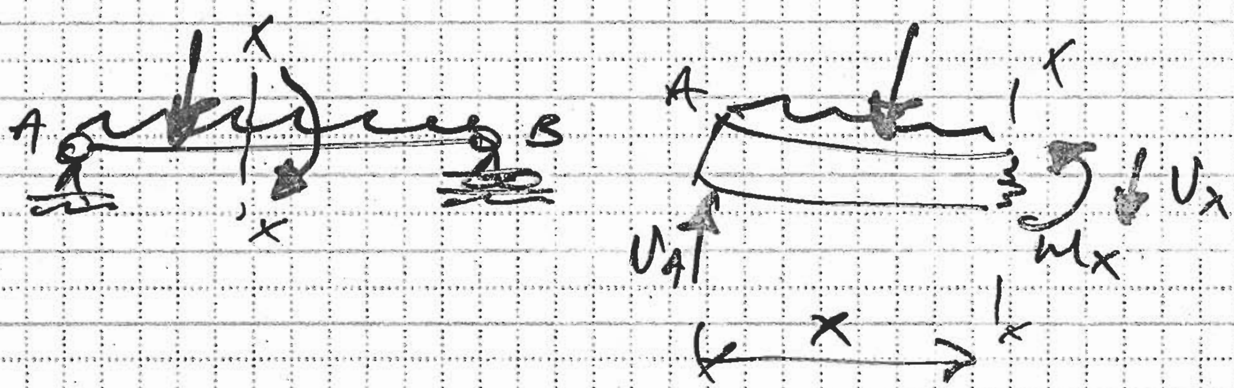
Direction of cut change

Each side resists an 'N' force system: up on left, down on right.

Variation of Shear & Moment

In the last example we found the shear & moment at two points, but in general we can take any point in the structure and can expect to get different results for V & M.

We are interested in how V & M change along the length of members:



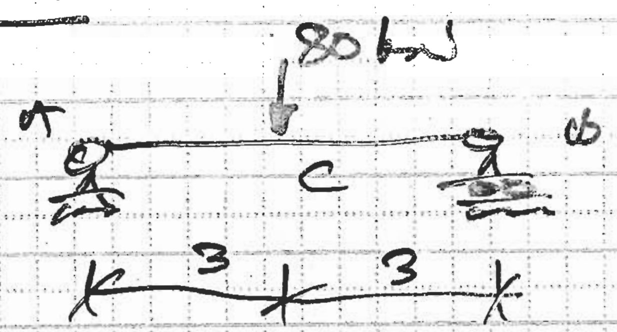
So as we vary the distance x we get different results for M_x & V_x.

The graphs of shear & moment against distance are called

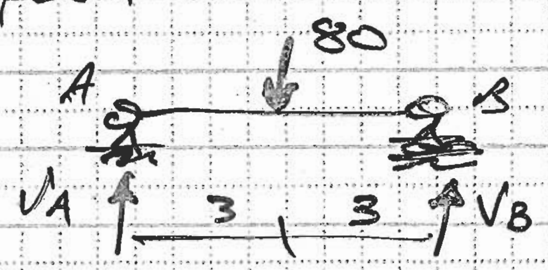
- Shear Force Diagram (SFD)
- Bending Moment Diagram (BMD)

respectively.

Example



First find the reactions:

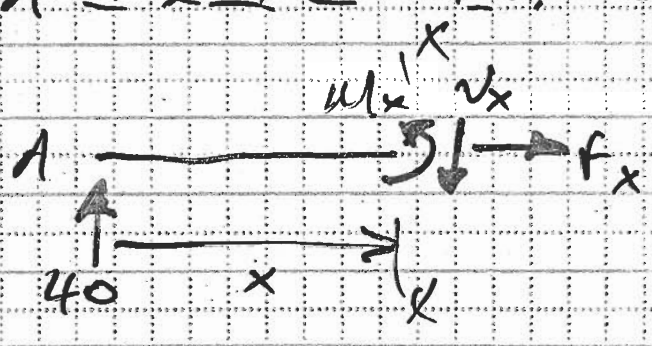


$\Sigma M_{about\ A} = 0 \therefore 80 \times 3 - 6V_B = 0 \therefore V_B = 40\,kN \uparrow$

$\Sigma F_y = 0 \therefore 80 - V_A - 40 = 0 \therefore V_A = 40\,kN \uparrow$

Since we have a point load, we must consider each portion separately

- $A \le x \le C$ i.e. $0 \le x \le 3$:



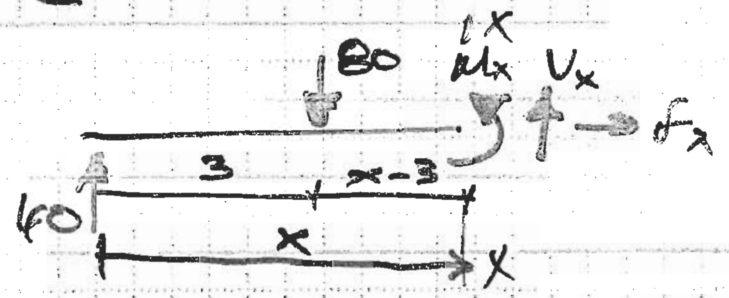
$\Sigma F_x = 0 \therefore F_x = 0$

$\Sigma F_y = 0 \therefore V_x = 40\,kN$

$\Sigma M_{about\ x-x} = 0 \therefore M_x - 40x = 0$

$\therefore M_x = 40x$

• $C \le x \le B$ i.e. $3 \le x \le 6$



$\Sigma F_x = 0 \therefore F_x = 0$

$\Sigma F_y = 0 \uparrow +$

$\therefore 40 - 80 + V_x = 0$

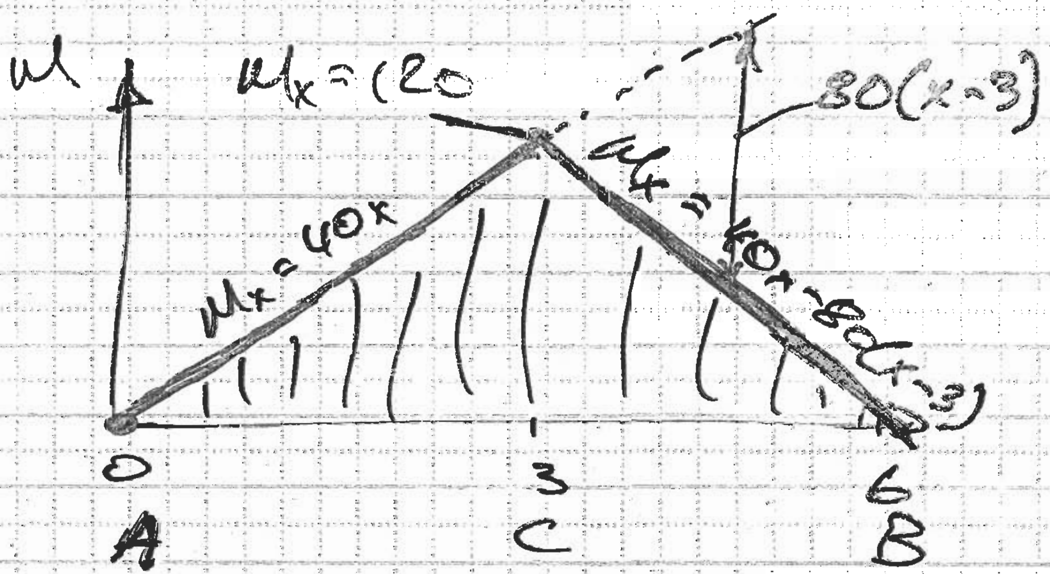
$\therefore V_x = 40\text{kN}$

ΣM about $x-x = 0$ $+\circlearrowright$

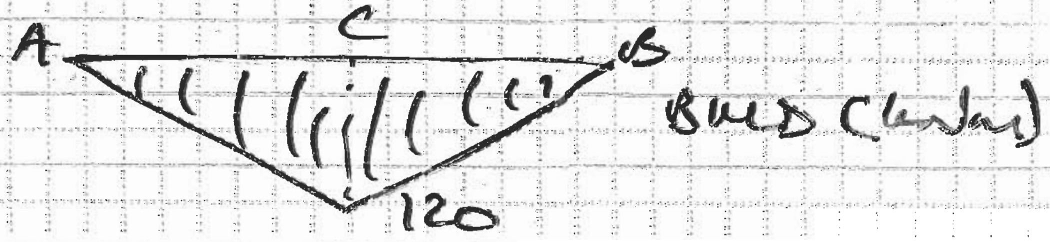
$\therefore M_x + 80(x-3) - 40x = 0$

$\therefore M_x = 40x - 80(x-3)$

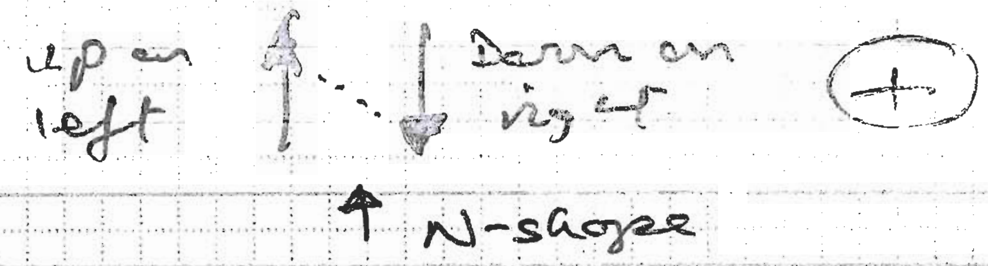
Plotting the equations for the moment:

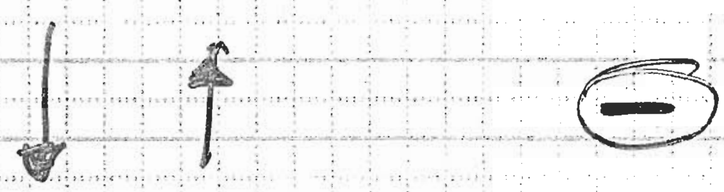


We usually (ok, always!) plot moments on the tension side of the beam. In this example tension is on the bottom of the beam, so we have:

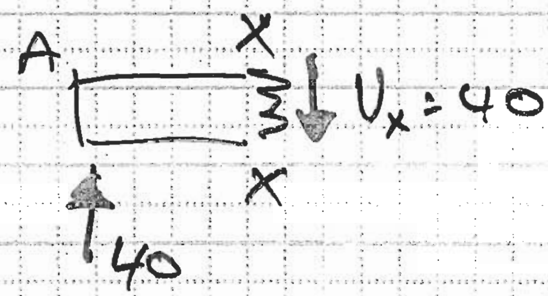


For shear we use the sign convention:



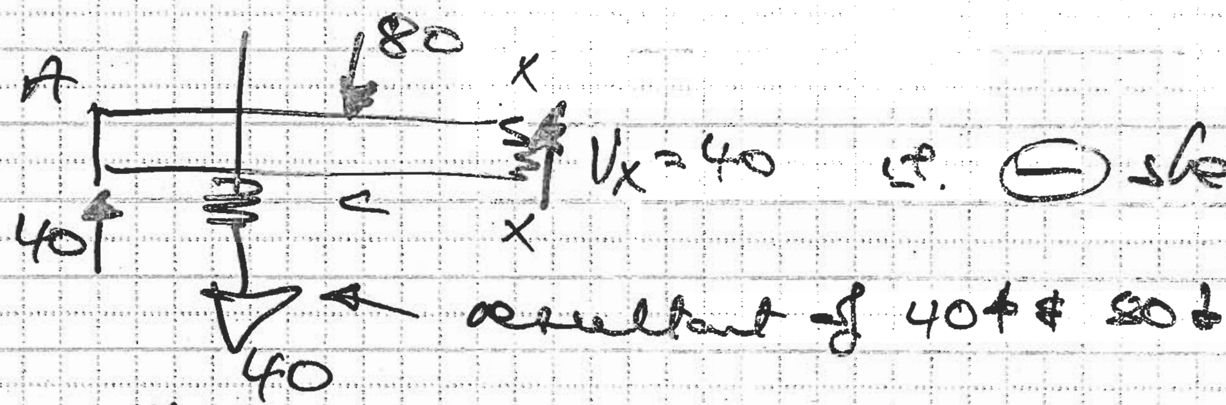


So for A to C, we have:



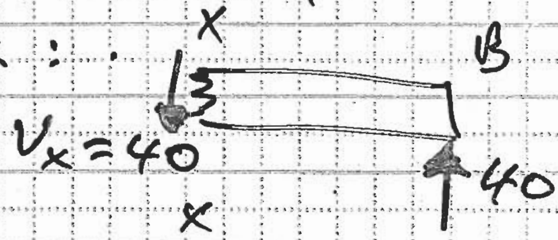
i.e. + shear

And for C to B:



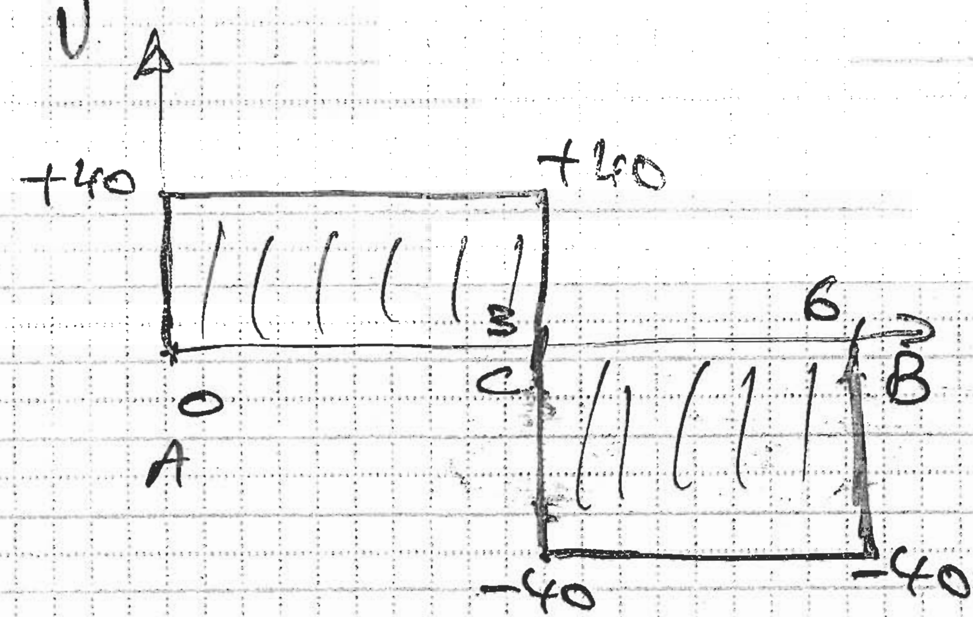
i.e. − shear

OR:

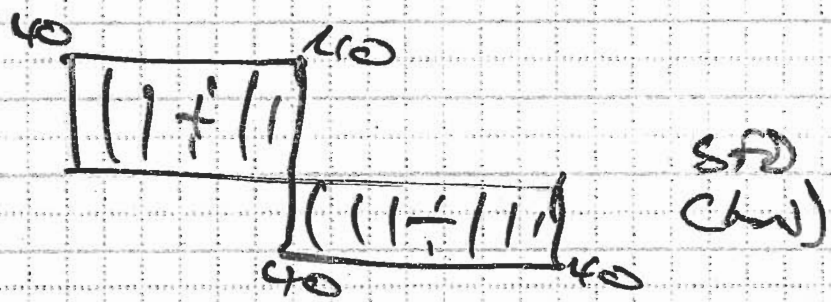


i.e. − shear.

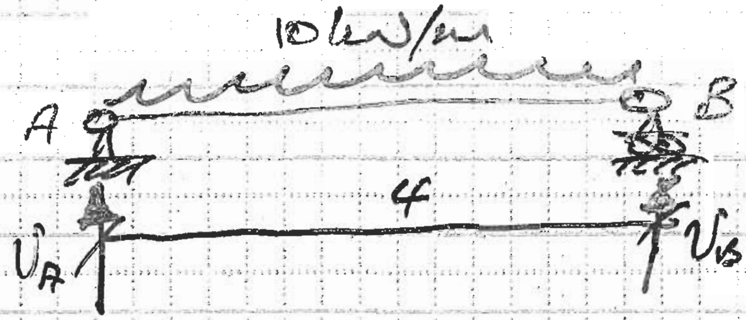
From the equations for shear force, we see that we have a constant value of 40 kN both sides of C:



Usually we don't draw the axes:



Example 2



- Reactions:

Σ M about $A = 0$

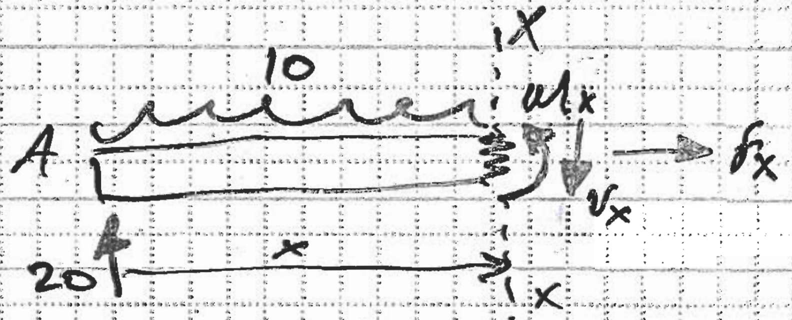
$(10\times4)(\frac{4}{2}) - 4V_B = 0 \therefore V_B = +20\text{ kN}$ i.e. ↑

$\Sigma F_y = 0$

$\therefore 10\times4 - V_B - V_A = 0 \therefore V_A = +20\text{ kN}$ i.e. ↑

- Internal stress resultants:

Draw the free body Diagrams showing <u>all</u> forces & moments that act:



Σ M about x-$x = 0 \therefore M_x + 10\frac{x^2}{2} - 20x = 0$

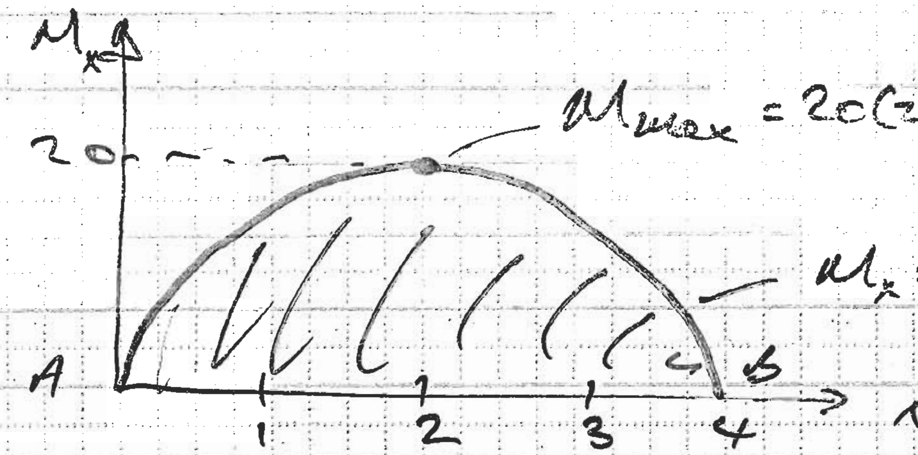
$\therefore M_x = 20x - 10\frac{x^2}{2}$

$\Sigma F_y = 0 \therefore 10x - 20 + V_x = 0$

$\therefore V_x = 20 - 10x$

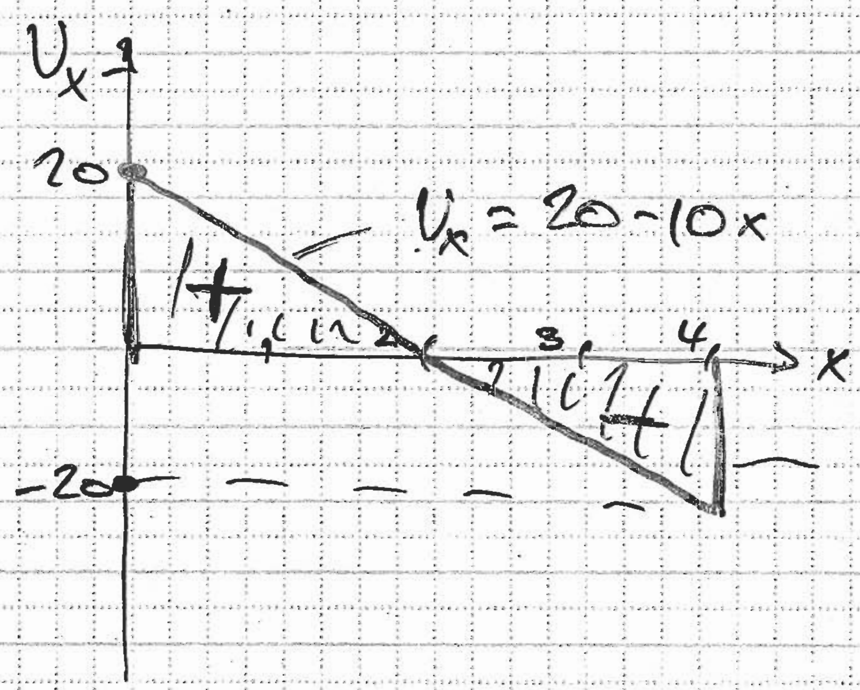
x is valid anywhere from A to B

i.e. from $0 \le x \le 4$.



$M_{max} = 20(2) - 10\frac{(2)^2}{2} = 20\,kNm$

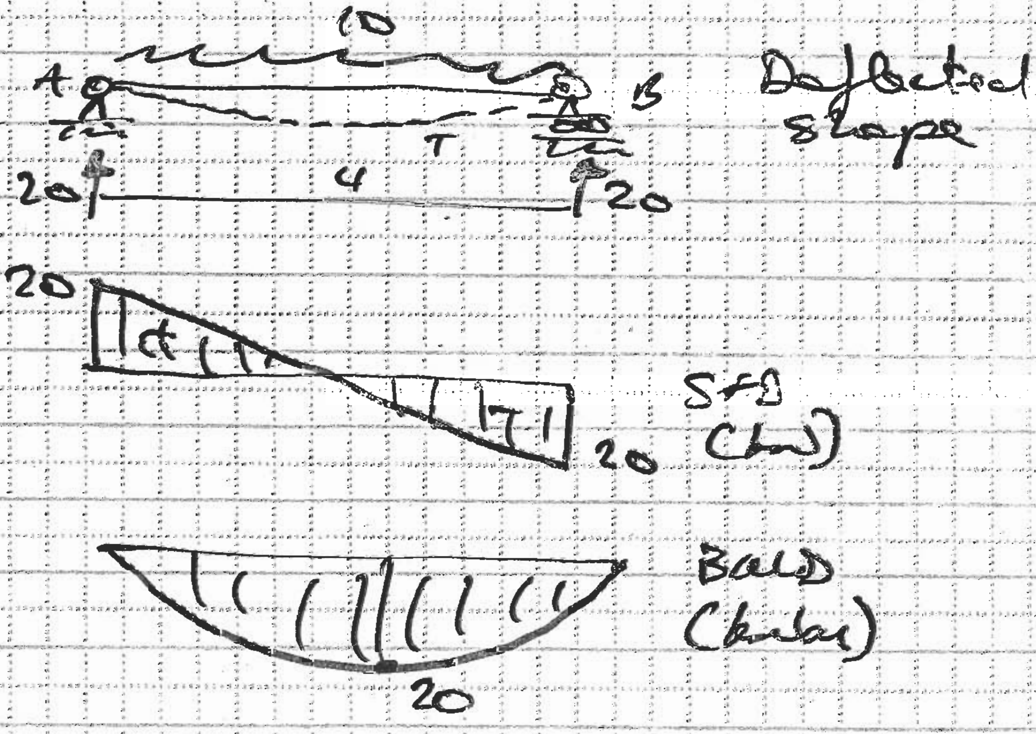
$M_x = 20x - 10\frac{x^2}{2}$



$V_x = 20 - 10x$

$V(4) = 20 - 10 \times 4$
$= -20\,kN$

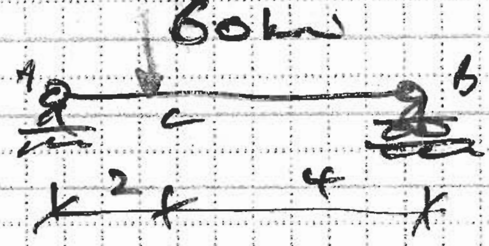
As usual though, we prefer to draw these diagrams as:



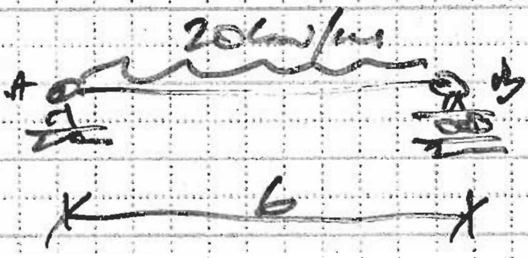
Problems

Draw the BMD & SFD and determine the reactions, for each of the following. Sketch also the deflected shape of the beam:

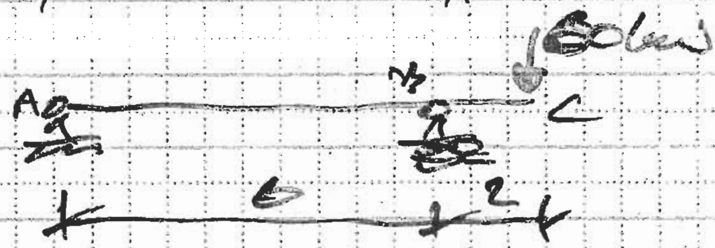
1.



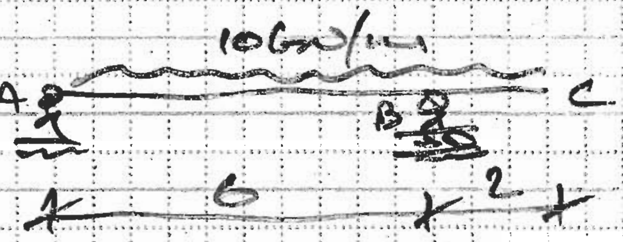
2.



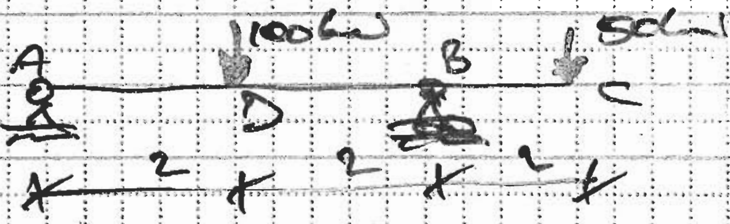
3.



4.

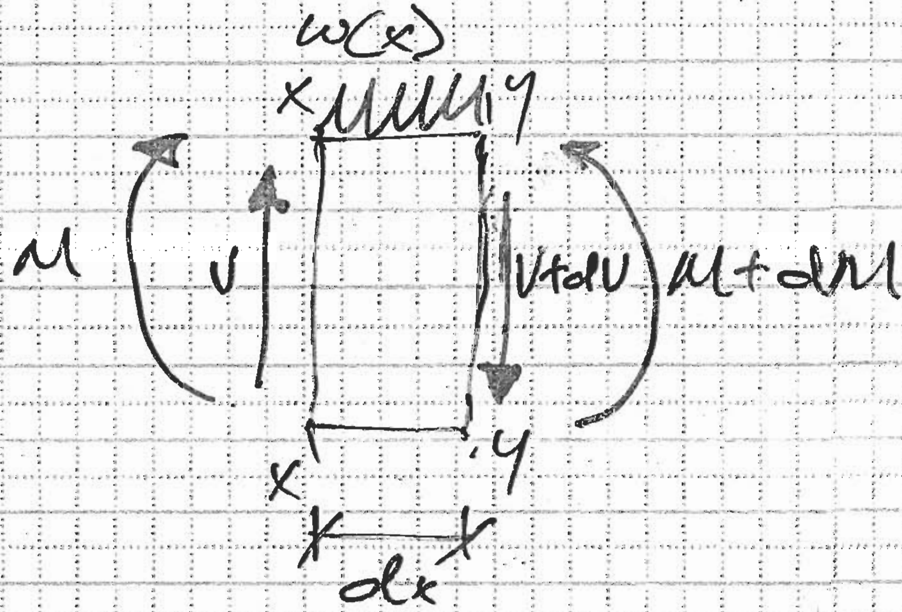


5.



The Relationship between Load, Shear and Moment

Considering a small portion of a beam:



dx - a small distance

dV - small change in shear

dM - small change in moment

w(x) - load as a function of distance.

Taking $\Sigma F_y = 0$ for this element, we have:

$$V - w(x)dx - (V + dV) = 0$$

$$-w(x)\,dx - dV = 0$$

$$\therefore \boxed{\frac{dV}{dx} = -w(x)}$$

So,

The rate of change of shear force is equal to the load ①

Taking moments about face x-x gives:

$$M - (M + dM) + (V + dU)dx + (w(x) \cdot dx) \cdot \frac{dx}{2} = 0$$

$$-dM + V \cdot dx + dU \cdot dx + w(x) \cdot \frac{(dx)^2}{2} = 0$$

We ignore terms which are the product of two infinitesimal quantities since they disappear:

$$dU \cdot dx \rightarrow 0$$

$$(dx)^2 \rightarrow 0$$

So, we have:

$$-dM + V \cdot dx = 0$$

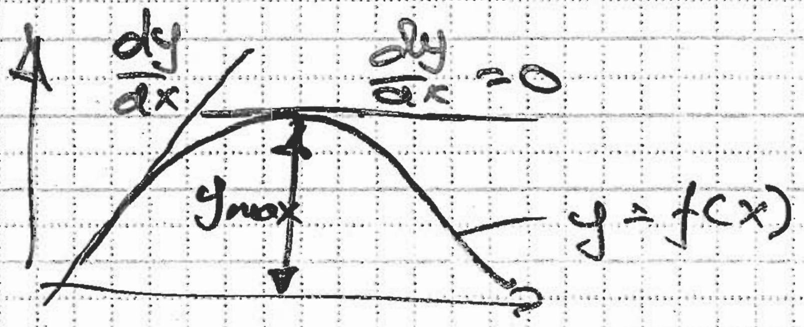
$$V dx = dM$$

$$\boxed{V = \frac{dM}{dx}}$$

Thus:

$\boxed{\text{Shear is the rate of change of bending moment}}$ (2)

From calculus, we know that the maximum value of a function $y = f(x)$ occurs where $\frac{dy}{dx} = 0$:



For us, we have:

$y \equiv$ moment, M

$\frac{dM}{dx} = 0$, when M is a maximum

But, $\frac{dM}{dx} = V$

So, when $V = 0$, M is a maximum

$\therefore$ $\boxed{\text{Maximum moment occurs at zero shear}}$ (3)

Returning to our first main equation:

$$\frac{dV}{dx} = w(x) \quad \text{(ignoring signs)}$$

Integrate both sides after multiply across

$$\int_A^B dV = \int_A^B w(x)\,dx$$

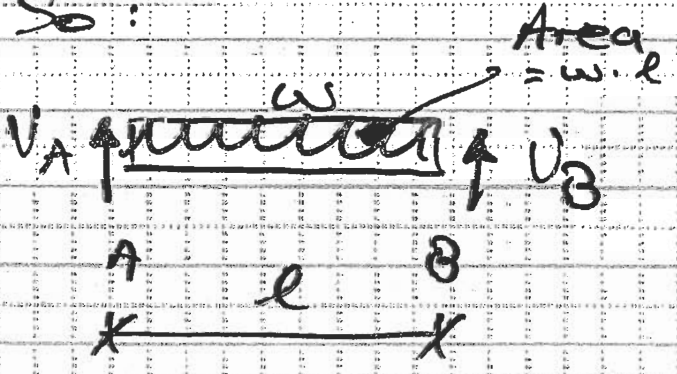
$$\therefore V_B - V_A = \text{Area of } w(x) \text{ diagram between B \& A}$$

$$\therefore \Delta V = \text{Area of load diagram.}$$

So,

The change in shear between two points is equal to the area of the load diagram between those points (4)

So:



$$V_B - V_A = w \cdot l$$

$$\text{So, } V_B = V_A + w \cdot l$$

Taking our second important equation:

$$V = \frac{dM}{dx}$$

Multiply across & integrate:

$$\int_A^B V\,dx = \int_A^B dM$$

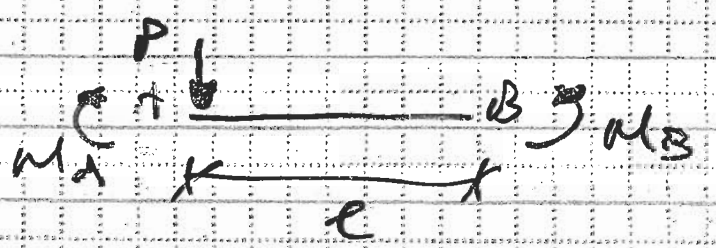
$\therefore$ $M_B - M_A$ = Area of shear diagram between A & B

$\therefore$ ΔM = Area of SFD between two points

So,

> The change in moment between two points is equal to the area of the SFD between those points (5)

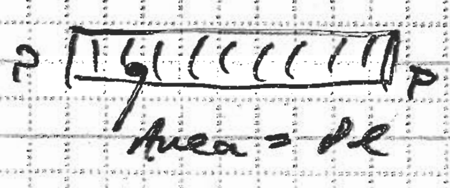
For example:



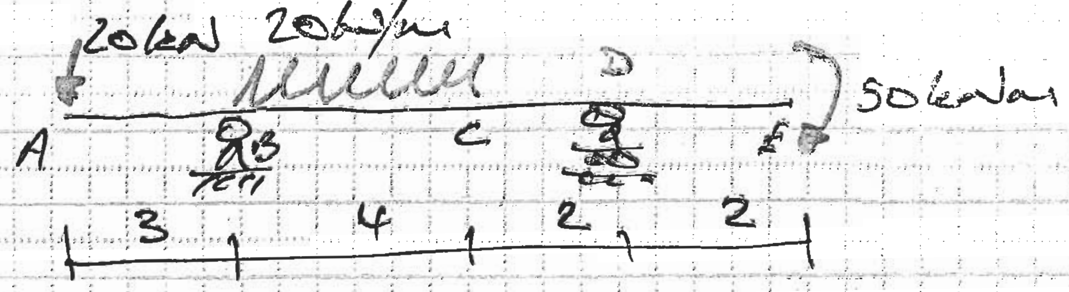
$M_B - M_A = P \cdot l$

$M_B = P \cdot l + M_A$

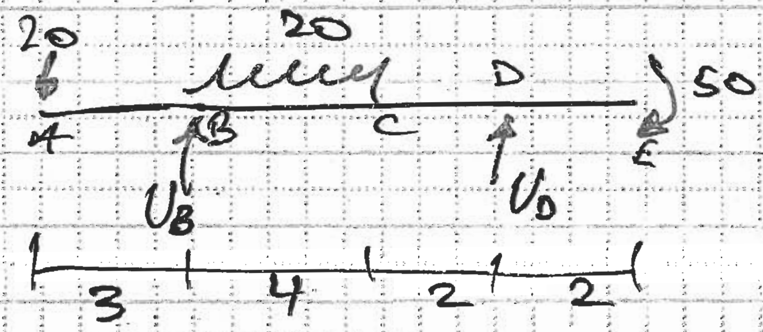
SFD



Example 3



Find BMD, Reactions, SFD & DSD



- Reactions:

ΣM about $B = 0$ ↻+

$$-20\times 3 + 20\times 4^2/2 - 6V_D + 50 = 0$$

$$V_D = +25\text{ kN} \text{ i.e. } \uparrow$$

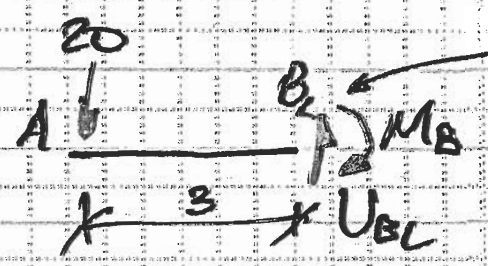
$\Sigma F_y = 0$ ↑+

$$V_B + 25 - 20 - 20\times 4 = 0$$

$$\therefore V_B = +75\text{ kN} \text{ i.e. } \uparrow$$

- FBDs:

$- |AB|_L$:



Arrow shown coming from top i.e. tension is assumed on top.

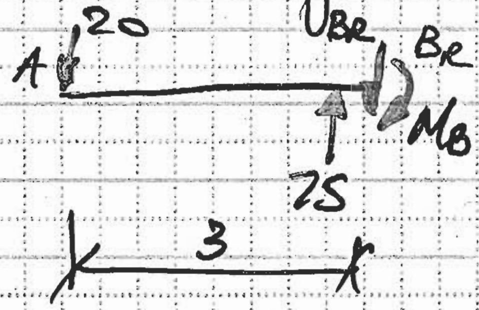
ΣM about $B = 0$ $\circlearrowright +$

$\therefore M_B - 20 \times 3 = 0 \quad \therefore M_B = 60\ kNm$

$\Sigma F_y = 0 \uparrow + \quad \therefore -20 + V_{BL} = 0 \therefore V_{BL} = 20\ kN \uparrow$

$\ominus$ shear

$- |AB|_R$

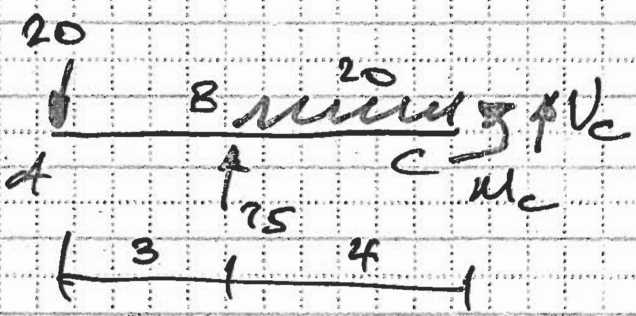


ΣM about $B = 0 \quad \therefore M_B = 20 \times 3 = 60\ kNm$

$\Sigma F_y = 0 \uparrow + \quad \therefore -20 + 75 - V_{BR} = 0$

$\therefore V_{BR} = 55\ kN$ i.e. $\oplus$ shear

$- |AC|$



ΣM about $C = 0$ $\circlearrowright +$

$\therefore M_C + 20 \times 4^2/2 + 20 \times 7 - 75 \times 4 = 0$

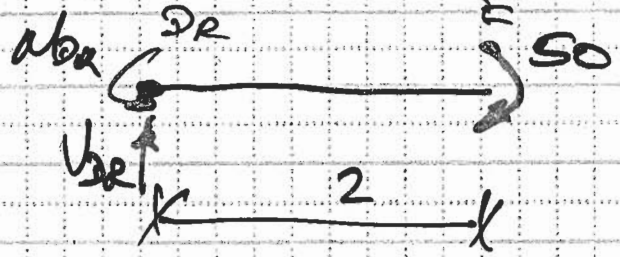
$\therefore M_C = 0\ kNm$

Thus C is a <u>point of contra flexure</u>, i.e. the point where moment changes the side of the member on which it is causing tension. In this case, tension at top is changing to tension on bottom of C.

$\Sigma F_y = 0$ ↑+ ∴ $20 - 75 + 20 \times 4 - V_C = 0$

∴ $V_C = +25$ kN ↑ so ⊖ shear, not ↑...↓

$-|D_R E|$



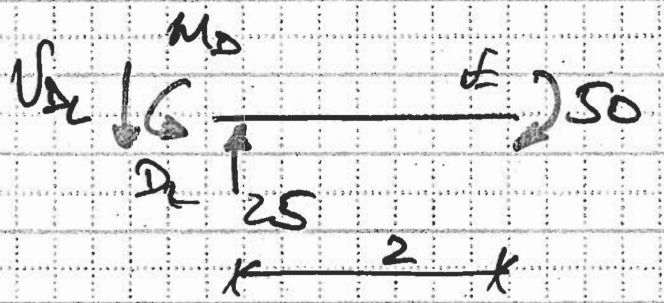
ΣM about $D_R = 0$

∴ $M_D = 50$ kNm

$\Sigma F_y = 0$

∴ $V_{DR} = 0$

$-|D_L E|$



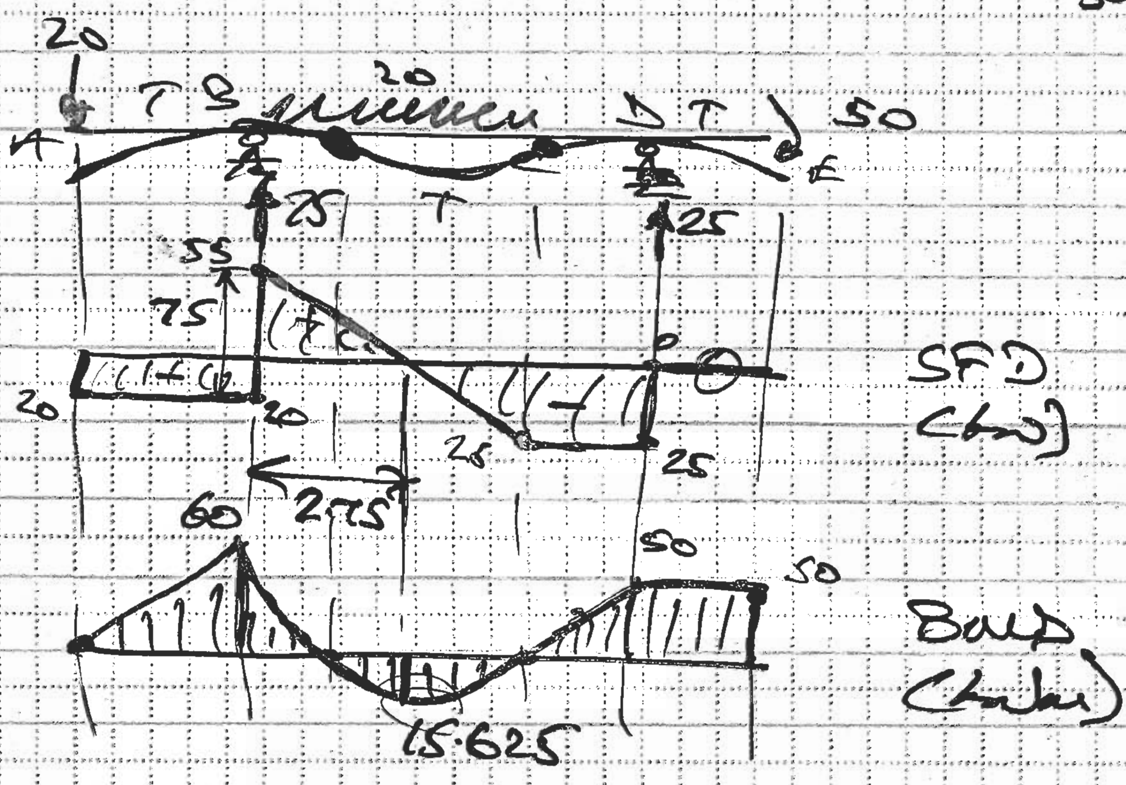
ΣM about $D_L = 0$

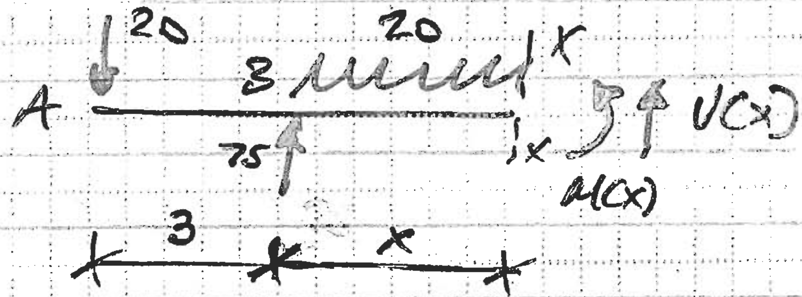
∴ $M_D = 50$ kNm

$\Sigma F_y = 0$

∴ $V_{DL} - 25 = 0$

∴ $V_{DL} = 25$ kN ↓ ⊖ shear





Σ Moment $x-x = 0$ +↻

$$\therefore\ M(x) + 20(3+x) + 20\frac{x^2}{2} - 75x = 0$$

$$M(x) = 75x - 20(3+x) - 20\frac{x^2}{2}$$

$$= 75x - 60 - 20x - 20x^2/2$$

$$M(x) = \underset{\text{moment at } B}{-60} + \underbrace{\underset{\text{shear at } B_R}{55x} - 20\frac{x^2}{2}}_{\text{Area of SFD from B to } x-x}$$

$\Sigma f_y = 0$ +↑

$$V(x) + 75 - 20 - 20x = 0$$

$$V(x) = \underbrace{20x}_{\text{shear changes by area of load diagram}} - \underbrace{55}_{\text{shear at } B_R}$$

We know max moment occurs at zero shear, $V=0$, so:

$V(x) = 20x - 55 = 0$

$\therefore \; x = \frac{55}{20} = \boxed{\frac{\text{start shear}}{\text{UDL}}}$

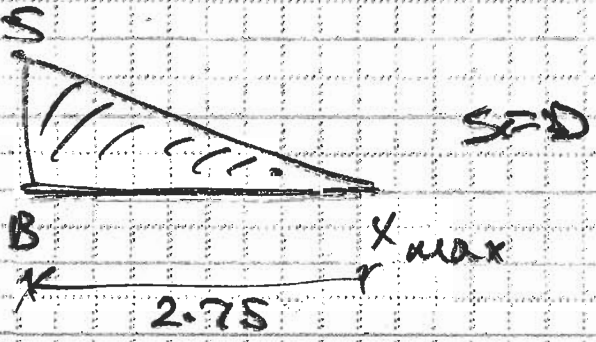
$= 2.75\,m$ from B.

To find the max moment, use this value for x in the equation for the moment:

$$M_{max} = -60 + 55(2.75) - 20\,\frac{2.75^2}{2}$$

Let's show that this is the area of the SFD:

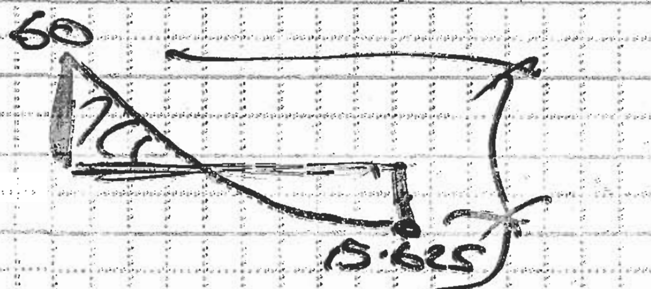
$$55 \cdot \left(\frac{55}{20}\right) - \frac{20}{2} \cdot \left(\frac{55}{20}\right)^2$$



$\Rightarrow \frac{55^2}{20} - \frac{1}{2} \cdot \frac{55^2}{20}$

$\Rightarrow \frac{1}{2} \cdot 55 \cdot \frac{55}{20}$

$\Rightarrow \frac{1}{2} \cdot (55) \cdot (2.75)$



$\Rightarrow$ Area of SFD

$= (75.625)$

ΔM = Area of SFD

So, $M_{max} = -60 + 75.625 = +15.625$ kNm

The ⊕ sign means tension on the bottom as assumed.

Lastly, we find the location of the point of contra flexure between B & C:

$M(x) = -60 + 55x - \frac{20}{2}x^2 = 0$ – for PoC

∴ Divide both sides by -10

$\therefore x^2 - 5.5x + 6 = 0$

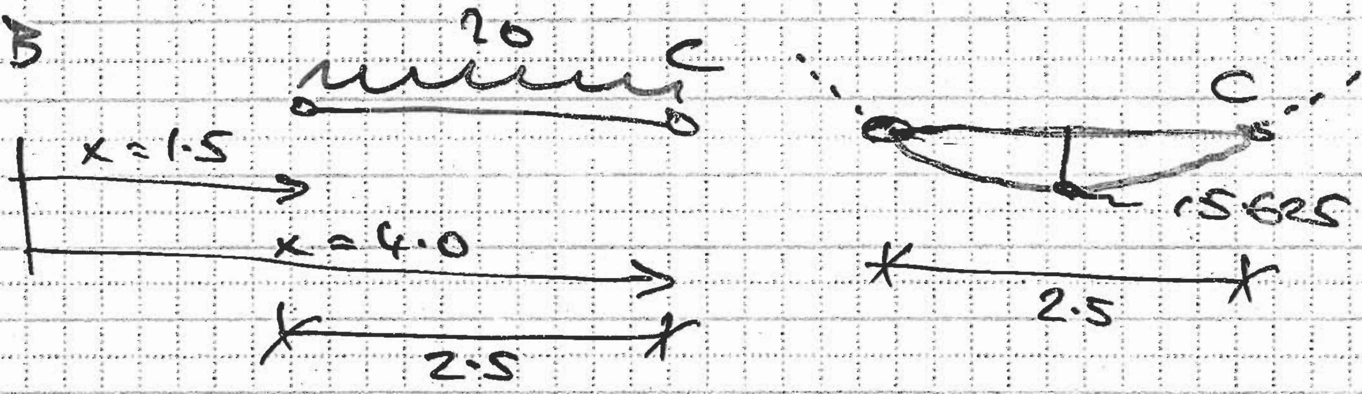
$$x = \frac{5.5 \pm \sqrt{5.5^2 - 4(6)}}{2}$$

$= 1.5\,m$ or $4.0\,m$

Because we are interested in the PoC between B & C, we choose the 1.5m distance.

Note the 4.0 m distance was expected, since we already knew the location of the 2nd PoC.

Lastly, since PoCs are effectively pins (since $M=0$), we can examine the portion between $x=1.5$ and $x=4.0m$ as if it were a small simply supported beam (only in terms of moment though):



Recalling $M = \frac{wl^2}{8}$ for an S.S. beam, we have:

$$M_{max} = \frac{20 \times 2.5^2}{8} = 15.625 \text{ kNm}$$

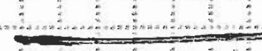
Finally, we see that the max moment occurs halfway between $x=1.5$ and $x=4.0$ since our little beam is symmetrical. That is M_{max} occurs at $x = 2.75m$ as previously found.

Structural Releases

As designers, we often want to dictate to a structure the way in which its stress resultants are distributed. We can introduce releases in a structure to effect this.

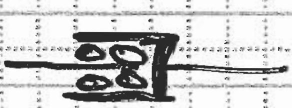
> A release is the removal of the ability of the structure to take a particular stress resultant.

The main release we will look at is the moment release, i.e. we 'release' the ability to take moment. We call this a hinge.

 – No release < shear, bending, axial

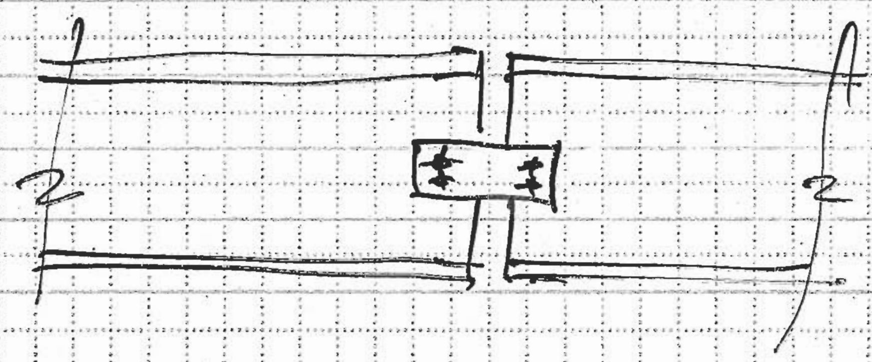
 – moment release (i.e. hinge) < shear, axial

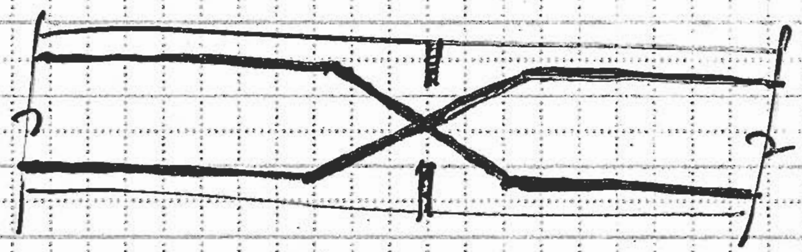
 – shear release < bending, axial

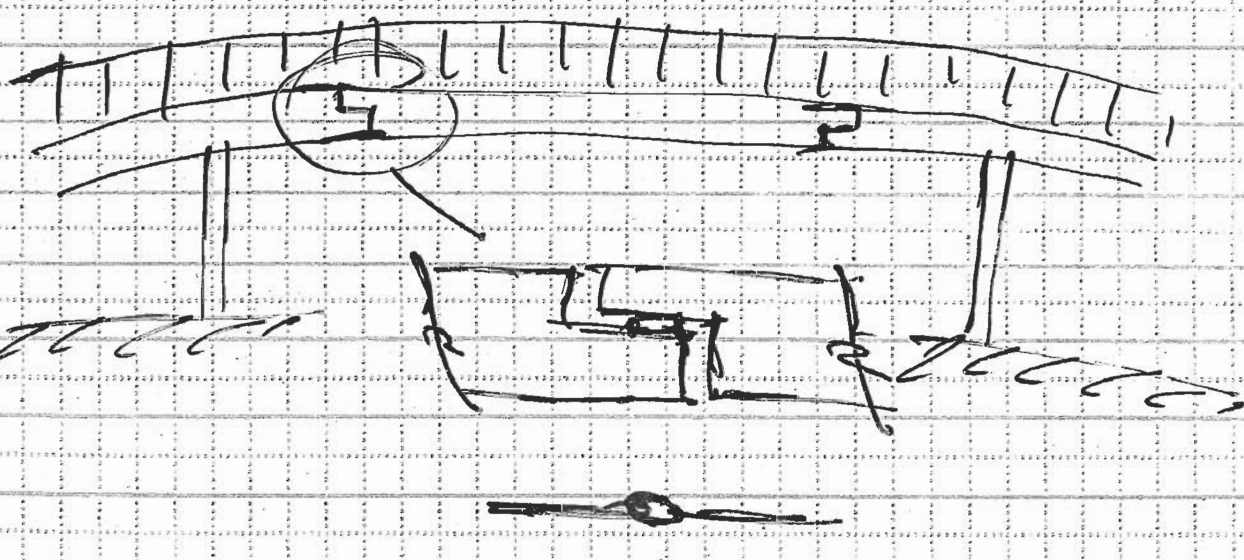
 – axial release < bending, shear

The introduction of a release gives us an extra 'knowledge' about the structure.
For example, with a hinge, we know the moment is zero - it cannot take bending moment.

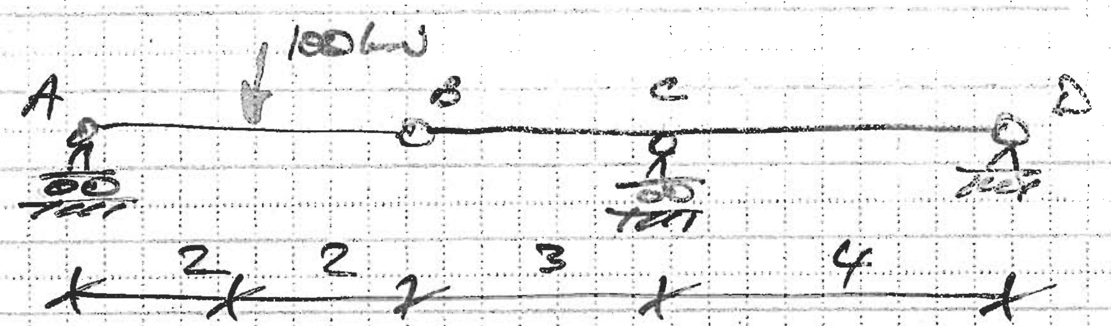
Examples:



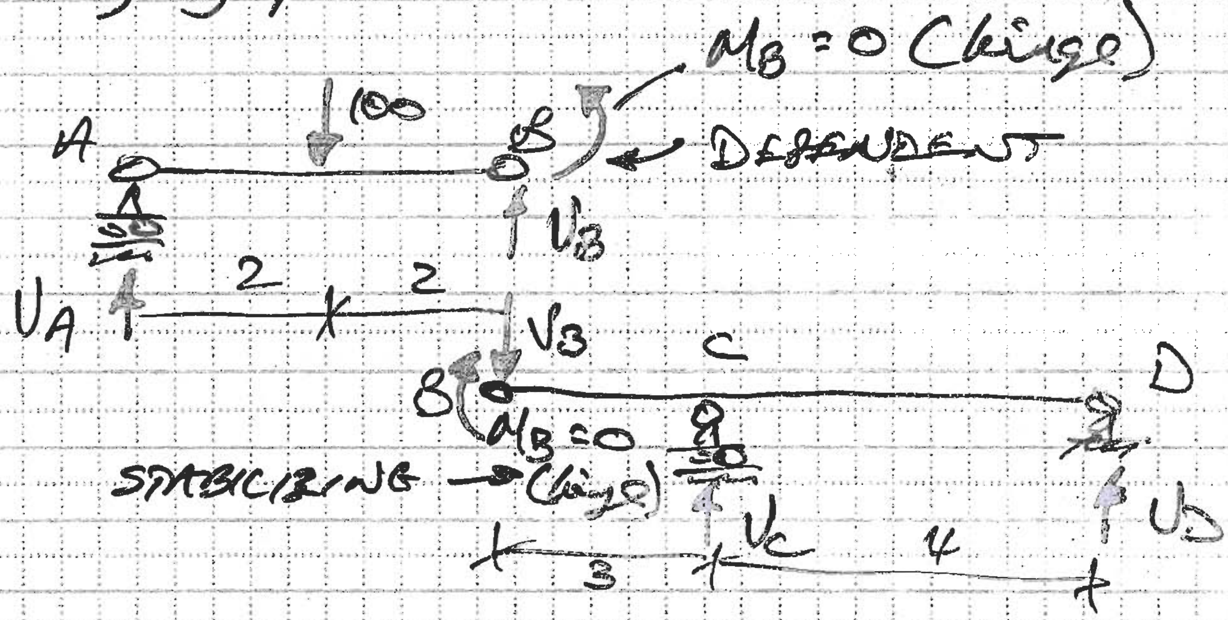




Example 1



To solve this we identify the dependent & stabilizing parts:



Note that even though there are 4 unknown reactions and only 3 eqns. of statics, we have the extra known of zero moment at the hinge, i.e. $M_B = 0$. Thus we can solve this structure.

For |AB|, the dependent structure, we take moments about B:

$M_B + 100 \times 2 - 4V_A = 0$

But $M_B = 0$ (hinge) $\therefore V_A = \frac{100 \times 2}{4} = 50\,kN \uparrow$

$\Sigma F_y = 0 \therefore 100 - V_A - V_B = 0 \therefore V_B = 50\,kN \uparrow$

For $[BD]$, Σ Moment $D = 0$

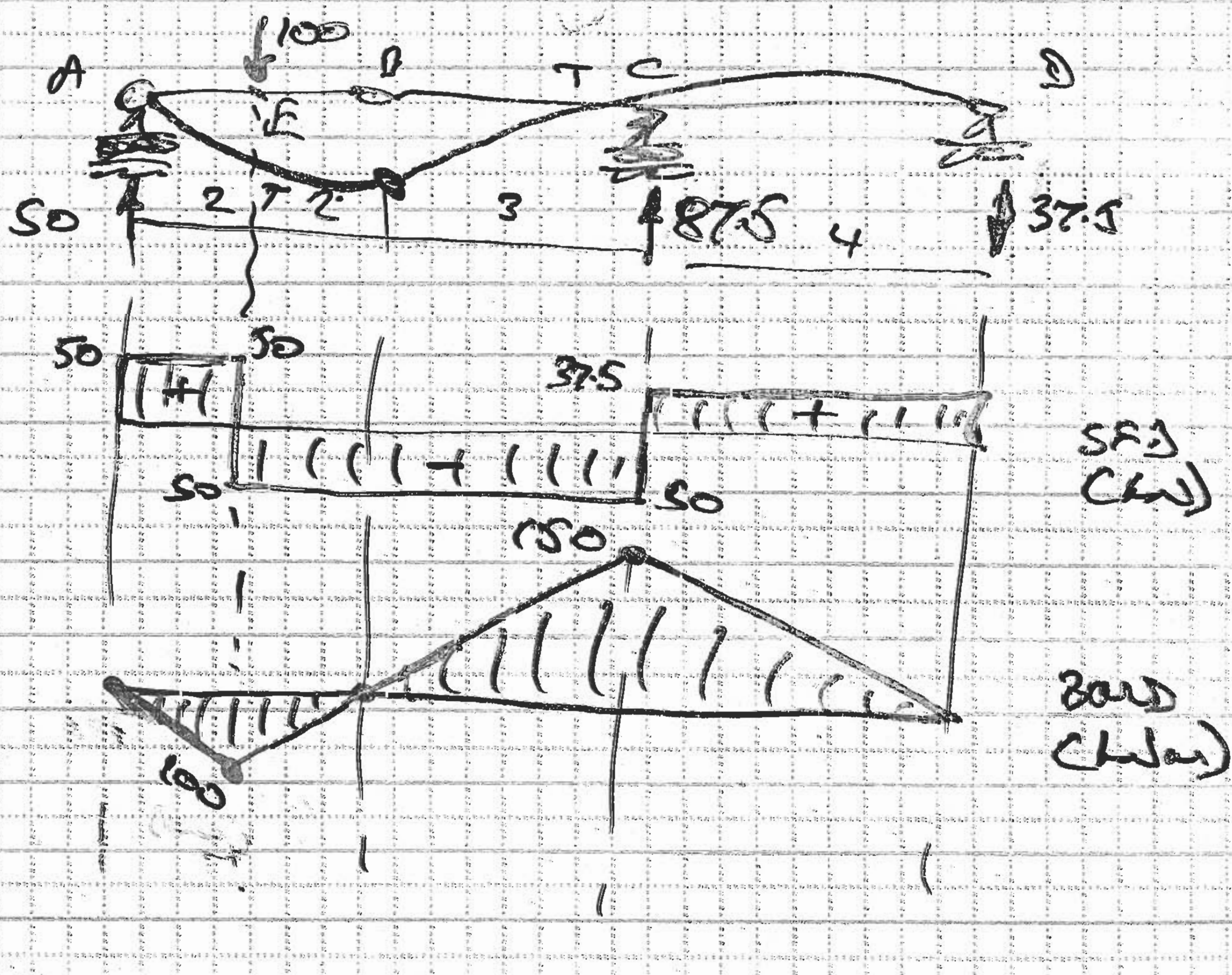
$50 \times 7 - 4V_C = 0$ (Note $M_B = 0$, hinge)

$V_C = 87.5\,kN \uparrow$

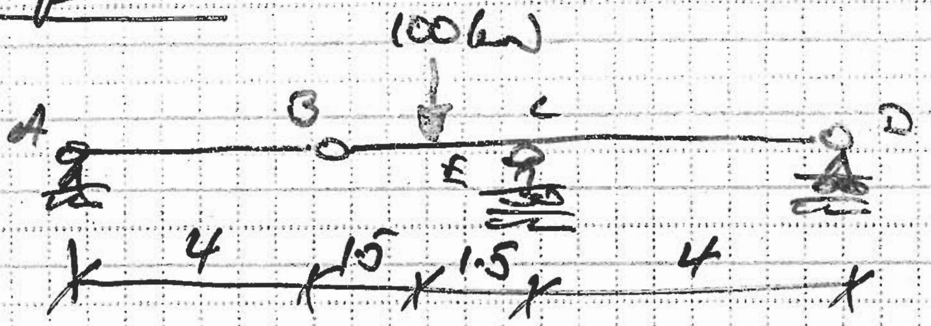
$\Sigma F_y = 0 \therefore V_B - V_C - V_D = 0$

$50 - 87.5 - V_D = 0$

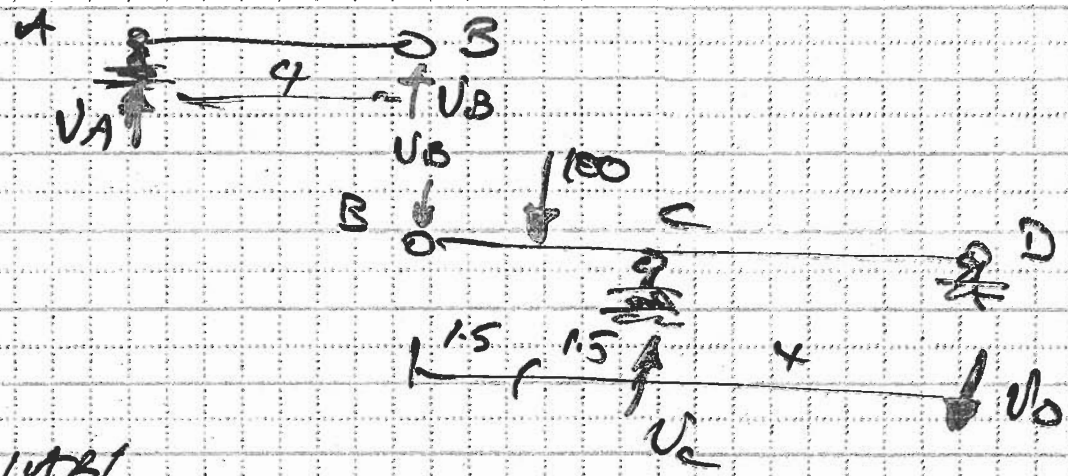
$\therefore V_D = -37.5\,kN$ i.e. $\downarrow$



Example 2



We split at the hinge as before:



For |AB|

$\Sigma M_{at} B = 0$: (remember, $M_B = 0$ - hinge)

$V_A \cdot 4 = 0 \quad \therefore V_A = 0$

$\Sigma F_y = 0 \quad \therefore V_B = 0$

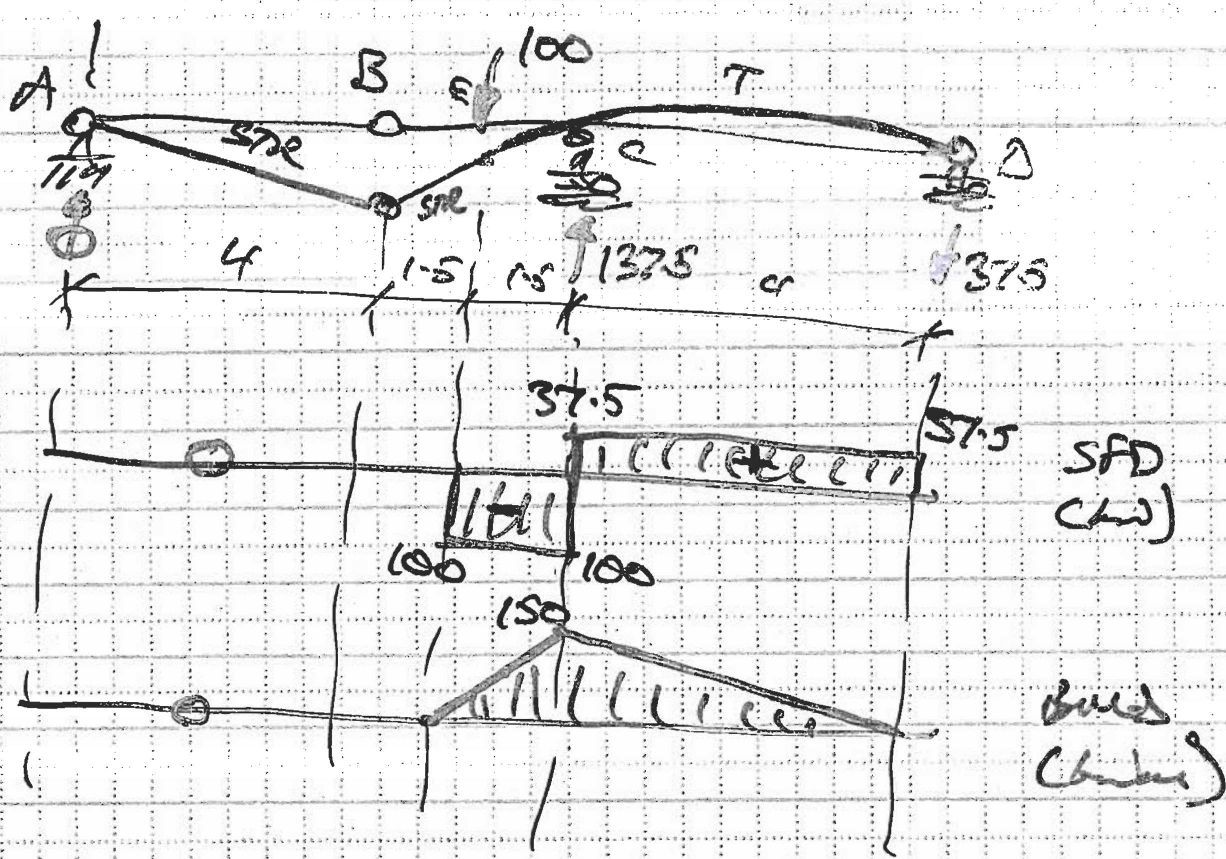
For |BD|

$\Sigma M_{at} D = 0$

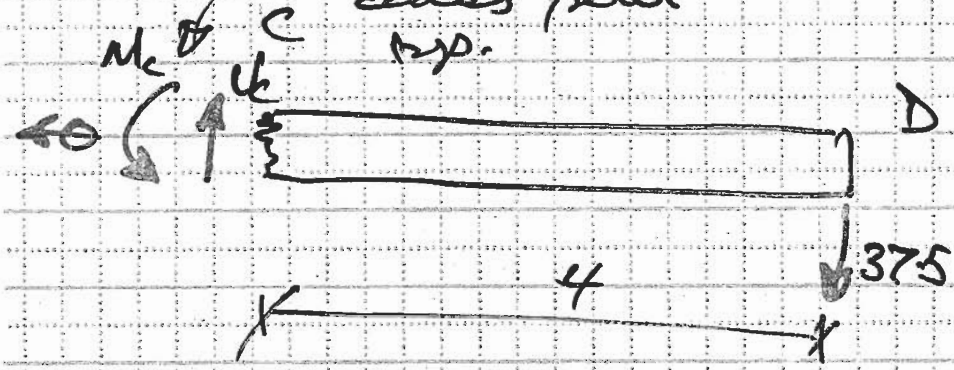
$\therefore 100 \times 5.5 - 4V_C = 0 \quad \therefore V_C = 137.5\,kN \uparrow$

$\Sigma F_y = 0 \quad \therefore 100 - 137.5 + V_D = 0$

$\therefore V_D = +37.5\,kN$ i.e. $\downarrow$



Assumes tension on top, since comes from top.



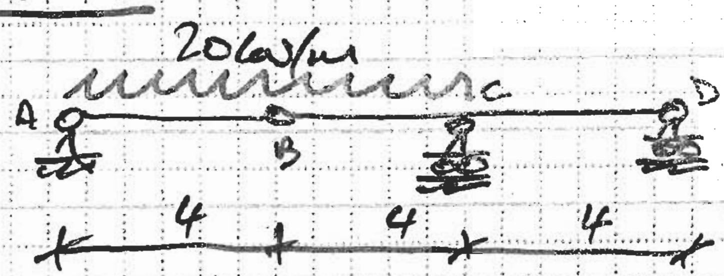
Σ M about C = 0

$\therefore M_c - 4 \times 37.5 = 0 \quad \therefore M_c = +150$ kNm

∴ Tension at top since ⊕ answer & I assumed tension on top of the start by drawing moment arrow coming from the top of the beam.

Problems

1



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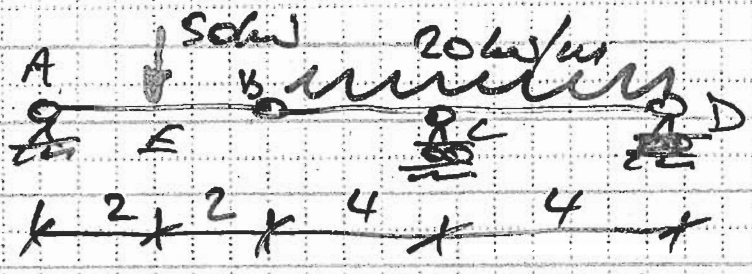

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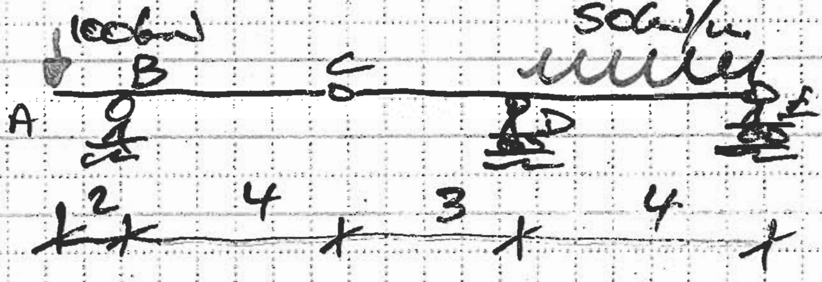

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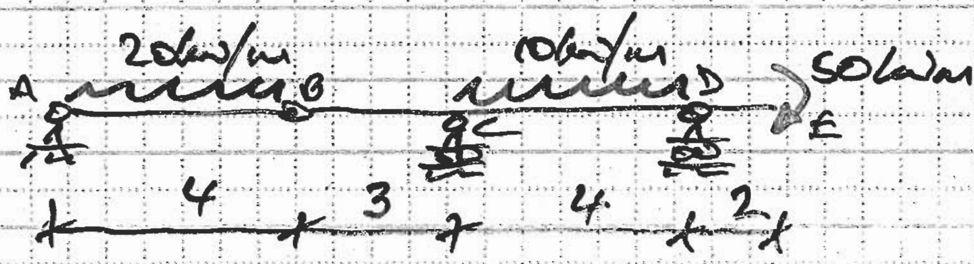

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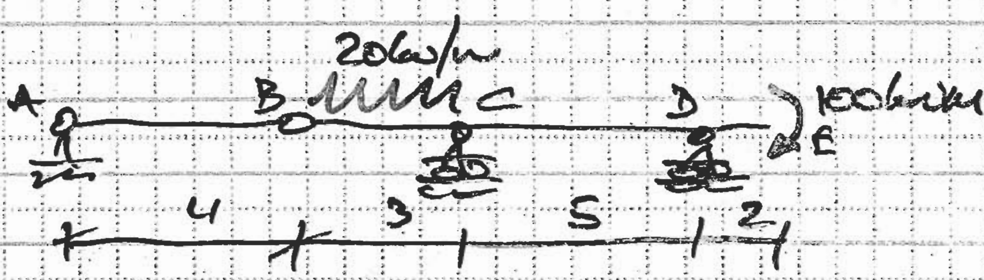

6